An ecological stock-flow-fund modelling framework

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Abstract: This paper develops an ecological stock-flow-fund (ESFF) modelling framework that provides an integrated platform for the analysis of the interactions between the ecosystem, the financial system and the macroeconomy. The ESFF framework has three distinct features. First, it formulates explicitly the monetary stocks and flows and the physical stocks and flows taking into account the accounting principles and the laws of thermodynamics. Second, it pays particular attention to the distinction between stock-flow resources and fund-service resources and formulates the interaction of funds with monetary and physical stocks and flows. Third, it allows an explicit analysis of the various stock-flow-fund channels through which the stability of the ecosystem, the financial system and the macroeconomy are interconnected. The paper illustrates how the ESFF framework can form the basis for the combined examination of ecological, financial and macroeconomic issues. In so doing, the suggested framework contributes to the development of an ecological macroeconomics in which finance plays a non-neutral role and physical and monetary variables are examined in an interconnected and consistent way.

Key words: Ecosystem, macroeconomy, finance, accounting principles, laws of thermodynamics, ecological stock-flow-fund modelling.

JEL codes: E1, E12, E44, Q57

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1. Introduction

Ecological macroeconomics is a new interdisciplinary field that examines the macroeconomy as part of the ecosystem, taking explicitly into account the biophysical limits of a finite planet (Harris, 2009; Jackson, 2011; Rezai et al., 2013; Jackson et al., 2014). It largely draws on the synthesis of ecological and post-Keynesian macroeconomics which has been identified as a fruitful avenue for the combined examination of economic and ecological issues (e.g. Holt and Spash, 2009; Mearman, 2009; Kronenberg, 2010; Spash and Ryan, 2012; Fontana and Sawyer, 2013, 2015).

The rationale for this synthesis can be briefly described as follows: Ecological economics provides a solid framework for the analysis of the economy-ecosystem interactions which is based on the conceptualisation of the economy as an open subsystem of the closed ecosystem as well as on the detailed analysis of the implications of the First and the Second Law of Thermodynamics. Post-Keynesian economics provides a quite rich explanation of the dynamics of modern capitalist economies by putting at the centre of its analysis the importance of aggregate demand, the non-neutral role of money and finance, the impact of fundamental uncertainty on economic decisions and the links between income distribution and economic activity. Ecological economics lacks the solid macroeconomic framework of post-Keynesian economics. Post-Keynesian economics almost totally ignores the ecological constraints of macroeconomic activity. Therefore, by synthesising these two fields, ecological macroeconomics can analyse in an integrated way the interactions between the ecosystem and the macroeconomy and can suggest new policies that are likely to attain a combination of macroeconomic stability and ecological sustainability.

Recent research has contributed to the development of the building blocks of ecological macroeconomics. Victor and Rosenbluth (2007), Victor (2012) and Barker et al. (2012) have presented large-scale models with Keynesian features that take into account the energy sector and various environmental issues. Jackson (2011), Fontana and Sawyer (2013), Rezai et al. (2013) and Taylor and Foley (2014) have put forward certain frameworks that combine
ecological with Keynesian (or post-Keynesian) insights. Godin (2012), Jackson et al. (2014), Berg et al. (2015), Naqvi (2015) and Fontana and Sawyer (2015) have examined environmental problems within stock-flow consistent or monetary circuit models that include a financial sector.

However, in the literature there is still a lack of an integrated framework that combines physical stocks, flows and funds and monetary stocks and flows in a consistent way. The development of such a consistent framework is important for the joint analysis of ecological issues (such as the depletion of ecosystem services and the degradation of natural resources), financial issues (such as financial fragility and the financing of green investment) and macroeconomic issues (such as growth and unemployment). This paper puts forward an ecological stock-flow consistent (ESFF) modelling framework that provides such an integrated analytical platform. Our modelling framework has three distinct features. First, it formulates explicitly the monetary stocks and flows and the physical stocks and flows taking into account the accounting principles and the laws of thermodynamics; the formulation of monetary stocks and flows draws on the stock-flow consistent literature (Godley and Lavoie, 2007); the formulation of physical stocks and flows draws on the tradition of Georgescu-Roegen (1971) and the literature on physical input-output tables (see e.g. Giljum and Hubacek, 2009). Second, the ESFF framework pays particular attention to the distinction between stock-flow resources and fund-service resources and formulates the interaction of funds with monetary and physical stocks and flows. The importance of funds in the examination of environmental issues has been pinpointed by Georgescu-Roegen (1971). Third, our framework allows an explicit analysis of the various stock-flow-fund channels through which the ecosystem, the financial system and the macroeconomy are interconnected.

The purpose of this paper is to describe the main features of this framework by presenting a benchmark ESFF model and by discussing the environmental, financial and macroeconomic issues that can be analysed by using this framework. The paper is organised as follows. Section 2 describes the main features of the ESFF modelling framework. Section 3 presents the matrices and the equations of a benchmark ESFF model. Section 4 outlines the main stock-flow-fund interactions between the ecosystem, the macroeconomy and the financial system. Section 5 concludes.
2. Main features of the ESFF modelling framework

Over the last decade or so, stock-flow consistent (SFC) modelling has become a very popular technique in post-Keynesian macroeconomics. Following the contributions of Godley (1999), Lavoie and Godley (2001-2) and Godley and Lavoie (2007), various post-Keynesian macroeconomists have used SFC models to analyse a plethora of macroeconomic issues. SFC models rely on the use of balance sheet and transactions matrices which allow the explicit consideration of the dynamic interactions between flows (e.g. interest, profits, wages) and stocks (e.g. loans, deposits, equities). The integration of accounting into dynamic macro modelling permits the detailed exploration of the links between the real and the financial spheres of the macroeconomy and illuminates the non-neutral role of money and finance.

However, a prominent drawback of the SFC models is the absence of any analysis of the transformation of matter and energy that takes place due to the economic processes. In SFC models it is assumed that the energy and matter that are necessary for production and consumption are available without limit and the degradation of ecosystem services has no feedback effects on the macroeconomy. Therefore, provided that there are no capital or labour constraints, the output of the economy is demand-determined and, hence, if an adequate policy mix is implemented, economic growth is theoretically feasible for an infinite period of time.

This feature comes in stark contrast with the fundamental propositions of ecological economists according to which that the macroeconomy is an open subsystem of the closed ecosystem and economic activity unavoidably respects the laws of thermodynamics. Ecological economists point out that, in line with the First Law of Thermodynamics, the macroeconomy continuously uses energy and matter inflows from the ecosystem and continuously produces waste which is an outflow to the ecosystem. Moreover, it is argued that by converting low-entropy materials and energy (e.g. fossil fuels and minerals) into high-entropy materials and energy (e.g. material waste and thermal energy), macroeconomic activities tend to increase the entropy in the

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1 See, for example, Le Heron and Mouakil (2008), Zezza (2008), van Treeck (2009), Dafermos (2012) and Nikolaidi (2014). See also Caverzasi and Godin (2014) for a recent review of the post-Keynesian stock-flow consistent modelling literature.
2 In thermodynamics, an open system is a system that exchanges both energy and matter with its surrounding environment. A closed system exchanges only energy, not matter. An isolated system exchanges neither energy nor matter.
3 For a presentation of the laws of thermodynamics and their implications for economics see e.g. Amir (1994), Baumgärtner (2002) and Daly and Farley (2011).
ecosystem. This stems from the Second Law of Thermodynamics and suggests that current economic activity reduces the ability of the macroeconomy to reproduce itself in the future.

Physical input-output accounting provides an adequate platform for the consideration of these ecological limits. Drawing on the flow-fund model of Georgescu-Roegen (1971, ch. 9; 1979; 1984) and the traditional Leontief monetary input-output tables, physical input-output accounting is a framework that records in a consistent way the physical flows associated with macroeconomic activities. In this framework the formulation of physical flows (uses of natural resources, emissions to nature etc.) relies explicitly on the First and the Second Law of Thermodynamics. Moreover, the energy-matter relationships between the various processes is depicted using input-output techniques that allow the researcher to analyse the use of resources and the amount of waste that correspond both to final and intermediate output. Importantly, in physical input-output tables all flows are measured in physical units (e.g. tonnes or tonnes of oil equivalent), avoiding thereby the problems that arise when market prices are assigned to natural resources.

The ESFF framework put forward in this paper is a synthesis of the post-Keynesian SFC models and the physical input-output framework. This synthesis allows us to combine the aforementioned advantages of these two approaches and to formulate the financial system and the macroeconomy as part of the ecosystem. Although the ESFF framework incorporates without modifications the transactions and the balance sheet matrices of the SFC framework, two important extensions are made in the physical input-output formulation.

First, based on the flow-fund model of Georgescu-Roegen a distinction is made between the stock-flow resources and the fund-service resources (see also Mayumi, 2001 and Daly and Farley, 2011). The stock-flow resources (energy and matter) are materially transformed into what they produce (including by-products), can theoretically be used at any rate desired and can be stockpiled for future use. The fund-service resources (labour, capital and Ricardian land) are not embodied in the output produced can be used only at specific rates and cannot be stockpiled for future use. The distinction between these two types of resources is significant basically for two reasons. First, it points out that production needs both fund-service and stock-flow resources, and there is no possibility to replace the one with the other. In conventional presentations of the production function this is not the case as capital, labour and natural

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4 For a detailed analysis of the features of physical input-output tables see Strassert (2002), Hoekstra and van den Bergh (2006) and Giljum and Hubacek (2009).
resources are described as ‘factors of production’ without a clarification of their different role; it is also argued that perfect substitutability is possible. Second, the distinction between stock-flow and fund-service resources is crucial for our understanding of biophysical limits: it emphasises that while people cannot use the services of fund resources at the rate that they wish, they can do so with the stock-flow resources. This implies that the stock-flow resources can be exhausted in a short period of time if people decide to increase substantially the associated flow rate.

Second, while the physical input-output framework focuses only on the physical flows, the ESFF analysis developed in this paper analyses also the dynamics of physical stocks that are considered to be the most important for economic activities. This is crucial because without the consideration of physical stocks it is not possible to explore issues of ecological sustainability. As explained below, this becomes possible by constructing a physical stock matrix.

Overall, the ESFF modelling framework relies on four matrices: 1) the physical input-output-fund matrix; 2) the physical stock matrix; 3) the transactions flow matrix; 4) the balance sheet matrix. The first matrix is an extension of the physical input-output tables. The second matrix represents selected stocks of matter and energy. The third and the fourth matrix describe the changes in the stocks and flows of the macroeconomic and financial systems, following the traditional formulations in the SFC literature.

3. Matrices and equations

The ESFF modelling framework will be presented by postulating a highly simplified structure of the economic processes. In particular, we consider a closed macroeconomy in which there is no government and no central bank. Firms make conventional and green investment by using retained profits, loans and equities. Households buy durable consumption goods. They do not take out loans. Commercial banks distribute all their profits to households. There is one type of material good in the economy that can be used for both (durable) consumption and investment purposes. Firms produce this good by using useful matter and useful (renewable and non-renewable) energy from the ecosystem, which have been previously transformed into a controlled form. Firms recycle a part of their waste which is then used as an additional inflow in the production of goods. In line with the post-Keynesian tradition, the production of goods is equal to their demand (consumption plus investment). However, this happens only when there are no supply constraints due to the unavailability of useful matter, useful energy, labour and
capital. If these constraints arise, the production is supply-constrained. We have explicitly considered this by postulating a Leontief-type production function. To avoid complications related to inflation, it is assumed that the price of the consumption and investment goods is equal to unity; this means that real values are equal to nominal values. However, the model considers financial inflation since the price of equities is allowed to fluctuate.

We will begin by analysing the physical stocks, the physical flows and the funds. Then, we will turn to monetary stocks and flows.

### 3.1 Physical stocks, flows and funds

Table 1 depicts the physical input-output-fund matrix. This matrix shows the transformations of matter and energy as a result of the economic processes. The matrix indicates flows of energy and matter that are measured either in mass units (e.g. tonnes) or in energetic units (e.g. tonnes of oil equivalent). The items that represent both mass and energy are measured in both mass and energetic units. Columns indicate the inputs that are necessary in each process as well as the by-products of the processes; the by-products (recyclable matter and emissions to nature) are denoted by a minus sign. The rows show how the outputs of the processes are used. The funds that are necessary for the economic processes (labour and capital) are presented at the bottom of the matrix.

The matrix captures the following processes:

1. **Production of controlled matter**: This is the process through which the useful matter that is in the ground (this could be, for example, silver, manganese or iron ore) is extracted and transformed in order to be used as an inflow in the production of goods and recycled matter. Controlled energy is necessary for this process to take place. The by-products of the process are the matter that can be recycled (this is used as an inflow in the fourth process) and the emissions to nature that include harmful matter (e.g. carbon emissions, chemical waste), non-harmful matter and dissipated energy (e.g. thermal energy).

2. **Production of controlled energy**: This process creates energy that can be used as an inflow in all economic processes. The energy is supplied either from renewable resources (e.g. solar, wind)

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5 In the related literature there is a debate about the way that the emissions to nature should be reported in physical input-output tables (see Hubacek and Giljum, 2003; Suh, 2004; Giljum et al., 2004; Dietzenbacher, 2005; Dietzenbacher et al., 2009). Here we adopt the approach that has been introduced by Suh (2004).

6 In the flow-fund model of Georgescu-Roegen (1971, ch. 9; 1979; 1984) Ricardian land is included in the funds. For simplicity, this fund is not incorporated in Table 1.
or non-renewable resources (e.g. oil, gas coal). Importantly, controlled energy is necessary as an inflow in this process. The emissions to nature include harmful matter, non-harmful matter and dissipated energy.

(3) Production of goods: This process produces goods for consumption and investment purposes using controlled energy, controlled matter and recycled matter. A part of the by-products is recycled. The rest is emitted to nature.

(4) Recycling: This process produces recycled matter using recyclable matter, controlled energy and controlled matter. Again, the by-products include harmful and non-harmful matter as well as dissipated energy.

Table 1: Physical input-output-fund matrix

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
<th>Intermediate output</th>
<th>Final output</th>
<th>Total output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Controlled matter</td>
<td></td>
<td>Controlled energy</td>
<td>Goods</td>
<td>TO1</td>
</tr>
<tr>
<td>Controlled energy</td>
<td></td>
<td>x21</td>
<td>x22</td>
<td>TO2</td>
</tr>
<tr>
<td>Material goods</td>
<td></td>
<td>x23</td>
<td>x24</td>
<td>TO3</td>
</tr>
<tr>
<td>Recycled matter</td>
<td></td>
<td>x43</td>
<td>x_f</td>
<td>TO4</td>
</tr>
</tbody>
</table>

Natural resources

Useful matter

Useful energy

Renewable energy

Non-renewable energy

Use of residuals

Recyclable matter

Supply of residuals

Recyclable matter

Emissions to nature

Dissipated matter

Harmful matter

Non-harmful matter

Dissipated energy

Total input

Funds:

Labour

Capital

<table>
<thead>
<tr>
<th></th>
<th>L1</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>UK_1</td>
<td>UK_2</td>
<td>UK_3</td>
<td>UK_4</td>
</tr>
</tbody>
</table>
The First Law of Thermodynamics suggests that energy and mass cannot be created or destroyed in the economic processes. Therefore, in each process the total inputs of mass should be equal to the total outputs of mass (mass balance) and the total inputs of energy should be equal to the total outputs of energy (energy balance). The Second Law of Thermodynamics is captured by the fact that the economic processes transform a part of useful energy and matter into dissipated energy and matter that are characterised by high entropy.

Table 2 depicts the physical stock matrix. This matrix refers only to the physical stocks that are considered to be the most important for economic activities. These include the non-renewable energy, the useful matter, the harmful matter and the material goods. Note that the stock of renewable resources is not represented in this matrix since the ability of nature to regenerate these resources implies that depletion issues are not very likely to emerge.

**Table 2: Physical stock matrix**

<table>
<thead>
<tr>
<th></th>
<th>Useful matter</th>
<th>Non-renewable energy</th>
<th>Harmful matter</th>
<th>Material goods</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Opening stock</strong></td>
<td>$M_{U,1}$</td>
<td>$E_{N,1}$</td>
<td>$M_{H,1}$</td>
<td>$M_{G,1}$</td>
</tr>
<tr>
<td><strong>Additions to stock</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discoveries of new stock</td>
<td>$+\text{DISC}_M$</td>
<td>$+\text{DISC}_E$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emissions to nature</td>
<td></td>
<td></td>
<td>$+\text{sh}$</td>
<td></td>
</tr>
<tr>
<td>Production of goods</td>
<td></td>
<td></td>
<td></td>
<td>$+x_f$</td>
</tr>
<tr>
<td><strong>Reductions to stock</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Extraction</td>
<td>$-m_1$</td>
<td>$-e_2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural dissipation</td>
<td></td>
<td></td>
<td>$-\text{ND}$</td>
<td></td>
</tr>
<tr>
<td>Destructions due to natural disasters</td>
<td></td>
<td></td>
<td></td>
<td>$-\text{DES}$</td>
</tr>
<tr>
<td><strong>Closing stock</strong></td>
<td>$M_U$</td>
<td>$E_N$</td>
<td>$M_H$</td>
<td>$M_G$</td>
</tr>
</tbody>
</table>

The first row of the matrix shows the stocks of the previous period. The last row presents the stock at the end of the current period after the additions and the reductions to stocks have taken place. The matrix indicates that the stock of non-renewable useful energy tends to reduce due to the extraction that is related to economic activity and tends to increase as a result of discoveries of new stocks. The same holds for the stock of useful matter. The stock of harmful matter tends to increase due to the emissions of the economic processes and tends to reduce as a result of natural dissipation. Lastly, the stock of material goods increases due to the economic

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7 For a similar presentation of the physical stocks see United Nations (2014).
8 It is assumed that the rate of creation of new natural resources by the nature is extremely low. Therefore, this inflow is assumed away.
production. However, we assume that there is a reduction in this stock whenever natural disasters take place.

Three physical ratios are of particular importance for the analysis of this paper. The first ratio is the depletion ratio of useful matter \( (dep_M) \), which is defined as the ratio of the extraction flow of useful matter \( (m_1) \) to its closing stock \( (M_U) \):

\[
dep_M = \frac{m_1}{M_U}
\]  

(1)

The second ratio is the depletion ratio of non-renewable energy \( (dep_E) \), which is also defined as the extraction flow \( (e_2) \) to its closing stock \( (E_N) \):

\[
dep_E = \frac{e_2}{E_N}
\]  

(2)

These two ratios capture the scarcity of useful matter and energy. The higher the extracted amount of matter and energy relative to the remaining stock, the higher the scarcity of these resources.

The third ratio is the degradation ratio \( (deg) \) that captures the degradation of ecosystem services that is caused due to the emission of harmful matter (greenhouse gas emissions and other types of waste); We assume that there is a threshold about the stock of the harmful matter \( (M_T) \). Beyond this threshold the ecosystem destabilises causing extreme environmental events. The degradation ratio is written as:

\[
deg = \frac{M_H}{M_T}
\]  

(3)

where \( M_H \) is the closing stock of harmful matter.

As the degradation ratio increases, the ecosystem’s absorption capacity deteriorates (see, e.g., Daly and Farley, 2011, ch. 6) and the climate change becomes more severe.
We proceed to describe the equations associated with the physical stocks, the physical flows and the funds. Note that the equations that capture identities derived from the matrices are denoted by using an ‘i’ next to the equation number.

### 3.1.1 Input-output relationships

Intermediate inputs are the inputs that one process demands from other processes. For example, \( x_{13} \) is the input that the goods production process demands from the controlled matter process. The inputs are determined as a proportion of the total output of each process. Algebraically:

\[
X = \delta \ TO
\]

where

\[
X = \begin{bmatrix}
0 & 0 & x_{13} & x_{14} \\
x_{21} & x_{22} & x_{23} & x_{24} \\
0 & 0 & 0 & 0 \\
0 & 0 & x_{43} & 0
\end{bmatrix}, \quad TO = \begin{bmatrix} TO_1 \\ TO_2 \\ TO_3 \\ TO_4 \end{bmatrix}, \quad \delta = \begin{bmatrix} 0 & 0 & \delta_{13} & \delta_{14} \\
\delta_{21} & \delta_{22} & \delta_{23} & \delta_{24} \\
0 & 0 & 0 & 0 \\
0 & 0 & \delta_{43} & 0 \end{bmatrix} \quad \text{and} \quad FO = \begin{bmatrix} 0 \\ 0 \\ 0 \\ x_f \end{bmatrix}.
\]

We have that \( x_{ij} \) is the intermediate input that process \( j \) demands from process \( i \), \( TO_i \) is the total output of process \( i \), \( \delta_{ij} > 0 \) are technical coefficients and \( x_f \) denotes the material goods that are produced for consumption and investment purposes (\( i, j = 1, 2, 3, 4 \)).

Note that in matrix algebra notation, a ‘hat’ over a vector denotes a diagonal matrix with the elements of the vector along the main diagonal. The total outputs are given by \( TO = (I - \delta)^{-1} FO \), where \( I - \delta \) is the Leontief inverse matrix and \( I \) is the 4x4 identity matrix (one in the main diagonal and zeros elsewhere).\(^9\)

The natural resources that the primary inputs in the production processes are the useful matter \( (m_i) \) and the useful energy \( (e_z) \). As in the case of intermediate inputs, these quantities are determined as a proportion of total output. In particular:

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\(^9\) For more details on the related matrix algebra see Miller and Blair (2009).
\[ m_1 = \theta_1 TO_1 \]  \hspace{1cm} (5)

\[ e_2 = \theta_2 TO_2 \]  \hspace{1cm} (6)

where \( \theta_i > 0 \ (i = 1, 2) \) are technical coefficients.

Useful energy is distinguished between renewable energy \( (er_2) \) and non-renewable energy \( (en_2) \). Renewable energy is a proportion \( (\phi) \) of total useful energy:

\[ er_2 = \phi e_2 \]  \hspace{1cm} (7)

Note that \( \phi \) is a technical coefficient.

Non-renewable energy is determined by Eq. (8):

\[ en_2 = e_2 - er_2 \]  \hspace{1cm} (8)

As mentioned above, each process produces certain by-products (recyclable matter and emissions to nature). These by-products are determined as a proportion of total output. Algebraically:

\[ \Gamma = \Lambda i = \psi TO \]  \hspace{1cm} (9)

where

\[
\Gamma = \begin{bmatrix} r_4 \\ s \\ d \end{bmatrix}, \quad \Lambda = \begin{bmatrix} w_1 & 0 & w_3 & 0 \\ s_1 & s_2 & s_3 & s_4 \\ d_1 & d_2 & d_3 & d_4 \end{bmatrix} \quad \text{and} \quad \psi = \begin{bmatrix} \psi_{11} & 0 & \psi_{13} & 0 \\ \psi_{21} & \psi_{22} & \psi_{23} & \psi_{24} \\ \psi_{31} & \psi_{32} & \psi_{33} & \psi_{34} \end{bmatrix}.
\]

We have that \( r_4 \) is the total recyclable matter, \( s \) is the total dissipated matter, \( d \) is the total dissipated energy, \( w_i \) is the recyclable matter produced by process \( i \), \( s_i \) is the dissipated matter produced by process \( i \), \( d_i \) is the dissipated energy produced by process \( i \) and \( \psi_{ij} > 0 \) are technical coefficients \( (i, j = 1, 2, 3, 4) \). In Eq. (9) we use \( i \) to denote a column vector of 1’s (of dimension 4).
Dissipated matter is distinguished between harmful dissipated matter \((s_h_i)\) and non-harmful dissipated matter \((s_{n_i})\). Harmful dissipated matter is a proportion \((\omega_i)\) of total dissipated matter:

\[
s_h_i = \omega_i d_i \quad (10)
\]

Note that \(\omega_i\) is a technical coefficient \((i = 1,2,3,4)\).

Non-harmful dissipated matter is determined as a residual:

\[
s_{n_i} = s_i - s_h_i \quad (11)
\]

In Table 1 all types of matter are measured in mass units. Renewable energy and dissipated energy are considered to be weightless and, therefore, they are measured only in energetic units. Non-renewable energy is measured in both mass and energetic units. Controlled energy is assumed to consist of a weightless and a non-weightless component. Therefore a proportion of controlled energy is measured only in energetic units, while the rest is also measured in mass units by using conversion factors from mass to energy. For simplicity, this proportion is considered to be equal to \(\phi\) (the proportion of renewable energy in total useful energy).

The mass and energy balances reflect the fact that in all economic process total outputs \((TO)\) are equal to total inputs \((TI)\) in energetic and mass units (namely \(TO_i = TI_i\) for \(i = 1,2,3,4\)). The balances are depicted following the methodology described in Liang et al. (2010). Using the dissipated matter as a residual, the mass balances are written as follows:

\[
s_i = x_{1i} + \zeta_{2i}(1-\phi)x_{2i} + x_{4j} + m_i + \zeta_{i}e_i + r_i - w_i - TO_i \quad (12i)
\]

In Eq. (12i) \(\zeta_{1}\) and \(\zeta_{2}\) are the conversion factors from energy to mass for useful energy and controlled energy respectively.

The energy balances for processes (1), (3) and (4) are shown in Eq. (13i):

\[
d_i = x_{2j} \quad (13i)
\]
where $i = 1, 3, 4$. 

For process (2) the energy balance is as follows:

$$
d_2 = e_n_i + e_r_i - x_{21} - x_{23} - x_{24}
$$

(14i)

### 3.1.2 Technical coefficients and efficiency

Technical coefficients determine the efficiency of the underlying processes.\(^\text{10}\) We make a distinction between three types of efficiency: (a) resource efficiency, (b) recycling efficiency and (c) pollution efficiency.

Resource efficiency is captured by the technical coefficients $\delta_{ij}$ and $\theta_i$, which measure the ratio of inputs to useful output. The lower these ratios the higher the efficiency. Of particular interest is the coefficient $\delta_{22}$ which measures the ratio of the energy input that is necessary to produce energy to the total energy produced in process (2). This coefficient captures the energy return on investment (EROI). Since $EROI = (TO_2 - x_{22})/x_{22}$, we have that $EROI = 1/\delta_{22} - 1$. It is assumed that resource efficiency increases when green capital becomes higher as a result of green investment (green investment will be discussed in section 3.2). Moreover, in line with the related empirical literature (see e.g. Hall et al., 2014), EROI is assumed to decline as the non-renewable energy resources reduce. The growth rate of $\delta_{ij}$ and $\theta_i$ is shown in Eqs. (15)-(17).

\[
\Delta \delta_{ij} / \delta_{ij} = f_{ij}(g_{G-1})
\]

(15)

\[
\Delta \delta_{22} / \delta_{22} = f_{ij}(g_{G-1}, dep_{E-1})
\]

(16)

\[
\Delta \theta_i / \theta_i = f_i(g_{G-1})
\]

(17)

In Eq. (15) $i, j = 1, 3, 4$ and in Eq. (17) $i = 1, 2$; $g_G$ is the growth rate of green capital.

---

\(^\text{10}\) For definitions of efficiency see, for example, Baumgärtner (2002) and Ayres and van den Bergh (2005).
Recycling efficiency is captured by the technical coefficients \( \psi_{1j} \), which are defined as the ratios of recyclable matter to total output in process (1) and process (3). Green investment improves recycling efficiency. Thus:

\[
\Delta \psi_{1j}/\psi_{1j} = f_{1j}(g_{G-1})
\]  

(18)

where \( j = 1,3 \).

Pollution efficiency is captured by the proportions of harmful matter in total dissipated matter. The higher these proportions the lower the pollution efficiency. Pollution efficiency improves when green investment takes place as well as when the ratio of renewable energy in total useful energy becomes higher (the latter is also affected by green investment). Therefore:

\[
\Delta \omega_{i}/\omega_{i} = f_{i}(g_{G-1}, \phi)
\]  

(19)

\[
\Delta \phi/\phi = f(g_{G-1})
\]  

(20)

where \( i = 1,2,3,4 \).

### 3.1.3 Funds

Of particular importance are the productivities of the funds in the economic processes (labour and capital). Labour productivity \( (\lambda) \) can be written as follows:

\[
\lambda_i = \varepsilon_i \chi_i \pi_i
\]  

(21)

where \( \varepsilon_i = \frac{\text{Total output}}{\text{Energy and matter inputs}} \), \( \chi_i = \frac{\text{Energy and matter inputs}}{\text{Hours per worker}} \) and \( \pi_i = \frac{\text{Hours per worker}}{\text{Number of workers}} \) for \( i = 1,2,3,4 \).

The ratio \( \varepsilon_i \) is determined by the technical coefficients \( \delta_{ij} \) and \( \theta_i \), the ratio \( \chi_i \) denotes the energy and matter inputs that the workers use per hour and the ratio \( \pi_i \) is determined by the
working hours arrangements. Note that $\xi_i \chi_i$ is the total output per labour hour. This expresses the service rate of labour. Eq. (22) postulates that the growth rate of $\chi_i$ is affected by the growth rate of conventional capital stock ($g_c$). It also expresses the idea that labour productivity is negatively affected by the degradation ratio which places upward pressures on the temperature of the earth and can adversely affect the health of the ecosystem, both of which tend to reduce productivity (see, for example, Dell et al., 2014).

$$\Delta \chi_i / \chi_i = f_i \left( \frac{-}{\text{deg}}, \frac{(+)}{g_c-1} \right)$$

(22)

Eq. (23) shows the number of workers per economic process:

$$L_i = \frac{TO_i}{\lambda_i}$$

(23)

Capital productivity can be expressed as:

$$\xi_i = \xi_i \eta_i v_i$$

(24)

where $\xi_i = \frac{\text{Total output}}{\text{Energy and matter inputs}}$, $\eta_i = \frac{\text{Energy and matter inputs}}{\text{Hours per capital unit}}$ and $v_i = \frac{\text{Hours per capital unit}}{\text{Capital units}}$ for $i = 1, 2, 3, 4$.

The ratio $\xi_i \eta_i$ expresses the service rate of capital. The utilised capital ($UK_i$) in each production process is determined according to Eq. (25).

$$UK_i = \frac{TO_i}{\xi_i}$$

(25)

3.1.4 Final output and the Leontief-type production function

The final output, in mass units, which is demanded for consumption and investment purposes ($x^D_j$) is given by:
\[ x_f^D = \mu Y \]  \hspace{1cm} (26)

where \( \mu \) denotes the mass units per unit of output (\( Y \)).

However, the actual output might be constrained due to the unavailability of energy, matter, labour and capital in at least one of the economic processes. Therefore, the actual final output in mass units is given by the minimum between the demanded final output (\( x_f^D \)) and the potential total one (\( x_f^* \)):

\[ x_f = \min(x_f^D, x_f^*) \]  \hspace{1cm} (27)

We have that:

\[ x_f^* = \min(x_{f1}^*, x_{f2}^*, x_{f3}^*, x_{f4}^*) \]  \hspace{1cm} (28)

where \( x_i^* \) is the potential final output that can be produced based on the potential total output \( TO_i^* \) \( (i = 1,2,3,4,) \). For the four economic processes we have that:

\[ x_{f1}^* = \frac{TO_1^*}{\delta_{13} + \delta_{14}\delta_{43}} \]  \hspace{1cm} (29)

\[ x_{f2}^* = \frac{TO_2^*(1 - \delta_{22})}{\delta_{21}(\delta_{13} + \delta_{14}\delta_{43}) + \delta_{23} + \delta_{24}\delta_{43}} \]  \hspace{1cm} (30)

\[ x_{f3}^* = TO_3^* \]  \hspace{1cm} (31)

\[ x_{f4}^* = \frac{TO_4^*}{\delta_{43}} \]  \hspace{1cm} (32)

The total output in each economic process can be restricted by the remaining stock of useful matter (\( M_{U_{i-1}} \)), the remaining stock of non-renewable energy (\( E_{N_{i-1}} \)), and the labour force and capital stock that are available in each process. For simplicity, it is postulated that the labour force in each process is a proportion of the total labour force (\( LFO \)): 
\[ LFO_i = \sigma_{ii} LFO \] \hspace{1cm} (33)

where \( \sum_{i=1}^{4} \sigma_{ii} = 1 \). Similarly, the total available stock in each process is given by:

\[ K_i = \sigma_{2i} K \] \hspace{1cm} (34)

where \( \sum_{i=1}^{4} \sigma_{2i} = 1 \).

We have that:

\[ TO_1^* = \min \left( \frac{M_{U-1}}{\theta_1}, \frac{(\delta_{21} + \delta_{14} \delta_{43}) E_{N-1} (1 - \delta_{22})}{\phi\theta_2 (\delta_{21} (\delta_{13} + \delta_{14} \delta_{43}) + \delta_{24} \delta_{43})}, LFO_1 \lambda_1, K_{1-\xi_1} \right) \] \hspace{1cm} (35)

\[ TO_2^* = \min \left( \frac{M_{U-1}}{\theta_1}, \frac{(\delta_{21} (\delta_{13} + \delta_{14} \delta_{43}) + \delta_{21} + \delta_{24} \delta_{43})}{\phi\theta_2 (\delta_{21} (\delta_{13} + \delta_{14} \delta_{43}) + \delta_{24} \delta_{43})}, \frac{E_{N-1}}{\theta_2}, LFO_2 \lambda_2, K_{2-\xi_2} \right) \] \hspace{1cm} (36)

\[ TO_3^* = \min \left( \frac{M_{U-1}}{\theta_1 (\delta_{13} + \delta_{14} \delta_{43})}, \frac{E_{N-1} (1 - \delta_{22})}{\phi\theta_2 (\delta_{21} (\delta_{13} + \delta_{14} \delta_{43}) + \delta_{24} \delta_{43})}, LFO_3 \lambda_3, K_{3-\xi_3} \right) \] \hspace{1cm} (37)

\[ TO_4^* = \min \left( \frac{\delta_{43} M_{U-1}}{\theta_1 (\delta_{13} + \delta_{14} \delta_{43})}, \frac{\delta_{43} E_{N-1} (1 - \delta_{22})}{\phi\theta_2 (\delta_{21} (\delta_{13} + \delta_{14} \delta_{43}) + \delta_{24} \delta_{43})}, LFO_4 \lambda_4, K_{4-\xi_4} \right) \] \hspace{1cm} (38)

### 3.1.5 Stock changes in energy and matter

Eqs. (39i)-(42i) show the identities that stem from Table 2. These identities show the changes in the stock of matter and energy that are of particular interest for the sustainability of economic processes.

\[ \Delta M_U = DISC_M - m_1 \] \hspace{1cm} (39i)

\[ \Delta E_N = DISC_E - e_2 \] \hspace{1cm} (40i)

\[ \Delta M_H = sh - ND \] \hspace{1cm} (41i)

\[ \Delta M_G = x_f - DES \] \hspace{1cm} (42i)
We have that $DISC_M$ is the stock of useful matter that is discovered every period, $DISC_E$ is the respective stock of non-renewable energy, $sh$ denotes the total harmful material emissions to nature ($sh = sh_1 + sh_2 + sh_3 + sh_4$), $ND$ is the natural dissipation of harmful matter due to ecosystem’s waste absorption capacity and $DES$ is the destruction of material goods due to natural disasters and $M_G$ is the stock of material goods.

It is postulated that the discoveries of new stocks are a proportion ($\tau_1, \tau_2 > 0$) of the remaining stocks. Thus:

$$DISC_M = \tau_1 M_{U-1} \quad (43)$$
$$DISC_E = \tau_2 E_{N-1} \quad (44)$$

Every period nature absorbs a specific fraction ($\tau_3 > 0$) of the stock of harmful matter:

$$ND = \tau_3 M_{H-1} \quad (45)$$

However, as mentioned above, the absorption capacity of the ecosystem reduces as the degradation ratio increases. This is reflected in Eq. (46):

$$\tau_3 = f\left(\frac{\text{deg}}{\text{f}}\right) \quad (46)$$

Finally, a proportion ($\tau_4 > 0$) of the stock of material goods is destructed due to disasters related to climate change. It is assumed that this destruction refers only to capital stock.$^{11}$

$$DES = \tau_4 MK_{-1} \quad (47)$$

where $MK$ is the capital stock expressed in mass units (note that $MK = \mu K$, where $K$ is the number of capital goods).

$^{11}$ See also Taylor and Foley (2014) and Naqvi (2015).
Since climate change becomes more severe as the degradation ratio increases, we assume that:

\[ \tau_4 = f \left( \frac{\text{deg}}{\text{deg}} \right) \]  

(48)

3.2 Monetary stocks and flows

We turn now to describe the monetary stocks and flows in the macroeconomy and the financial system. Table 3 is the transactions flow matrix of our economy. This matrix shows the transactions that take place between the various sectors of the economy (each row represents a transaction). For each sector inflows are denoted by a plus sign and outflows are denoted by a minus sign. The upper part of the matrix shows transactions that are related with the revenues and expenditures of the various sectors. The bottom part of the matrix indicates changes in financial assets and liabilities that arise from transactions. The columns represent the budget constraints of the sectors. For firms and commercial banks a distinction is made between current and capital accounts. The current accounts register payments made or received. The current accounts show the changes in assets and liabilities as well as the funds that are used to finance investment (in the case of firms). At the aggregate level, monetary inflows are equal to monetary outflows.

The economic activities of the sectors of the macroeconomy are summarised as follows:

(1) **Firms:** They conduct the 4 economic processes, described in section 3.1, which are necessary for the production of material goods. They invest both in conventional and green capital. They acquire external finance from equities and bank loans. The firm sector has not been disaggregated, which means that the monetary transactions between the 4 economic processes are netted out in our model.\(^\text{12}\)

(2) **Households:** They provide their labour services to firms and receive labour income. They also receive the distributed profits of firms, the profits of banks and the interest on deposits. Their income not consumed is saved in the form of deposits and equities. For simplicity, there is no household heterogeneity.\(^\text{13}\)

\(^\text{12}\) However, future extensions of the model could incorporate such a disaggregation which is particularly important because it allows the explicit formulation of the role of the prices of raw materials and energy. For stock-flow consistent models with ecological aspects that make such a disaggregation see Berg et al. (2015) and Naqvi (2015).

\(^\text{13}\) However, future extensions of the model could introduce different types of household in order to formulate the distribution of income. For a stock-flow consistent model with household heterogeneity see Dafermos and Papatheodorou (2015).
(3) **Banks:** They provide green and conventional loans to firms. Since they impose credit rationing, they provide only a proportion of the loans that are demanded. Deposits are their only liability.

**Table 3: Transactions flow matrix**

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
<th>Commercial banks</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>Capital</td>
<td>Current</td>
<td>Capital</td>
</tr>
<tr>
<td>Consumption</td>
<td>-C</td>
<td>+C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Conventional investment</td>
<td>+IC</td>
<td>-IC</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Green investment</td>
<td>+IC</td>
<td>-IC</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wages</td>
<td>+wL_{i,t}</td>
<td>-wL_{i,t}</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Firms' profits</td>
<td>+DP</td>
<td>-TP</td>
<td>+RP</td>
<td>0</td>
</tr>
<tr>
<td>Commercial banks' profits</td>
<td>+BP</td>
<td>0</td>
<td>-BP</td>
<td>0</td>
</tr>
<tr>
<td>Interest on deposits</td>
<td>+i_D D_{i,t}</td>
<td>-i_D D_{i,t}</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Destruction of conventional capital</td>
<td>-DES_C</td>
<td>+DES_C</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Destruction of green capital</td>
<td>-DES_G</td>
<td>+DES_G</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Interest on conventional loans</td>
<td>-i_C LC_{i,t}</td>
<td>+i_C LC_{i,t}</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Interest on green loans</td>
<td>-i_G LG_{i,t}</td>
<td>+i_G LG_{i,t}</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Δdeposits</td>
<td>-ΔD</td>
<td>+ΔD</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Δequities</td>
<td>-p,Δe</td>
<td>+p,Δe</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Δconventional loans</td>
<td>+ΔLC</td>
<td>-ΔLC</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Δgreen loans</td>
<td>+ΔLG</td>
<td>-ΔLG</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4 shows the assets and the liabilities of the sectors. We use a plus sign for the assets and a minus sign for the liabilities. Household and firms have non-zero net worth. Commercial banks have a zero net worth due to our assumption that they distribute all their profits. At the aggregate level, the net worth of the economy is equal to the capital stock of firms and the durable consumption goods of households which are the only real assets.
Table 4: Balance sheet matrix

<table>
<thead>
<tr>
<th></th>
<th>Households</th>
<th>Firms</th>
<th>Commercial banks</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>+K</td>
<td></td>
<td></td>
<td>+K</td>
</tr>
<tr>
<td>Durable consumption goods</td>
<td>+DC</td>
<td></td>
<td></td>
<td>+DC</td>
</tr>
<tr>
<td>Deposits</td>
<td>+D</td>
<td>-D</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Equities</td>
<td>+p_e</td>
<td>-p_e</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Conventional loans</td>
<td>-LC</td>
<td>+LC</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Green loans</td>
<td>-LG</td>
<td>+LG</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total (net worth)</td>
<td>+V_H</td>
<td>+V_F</td>
<td>0</td>
<td>+K+DC</td>
</tr>
</tbody>
</table>

The equations that are related to firms, households and banks are presented below. Note that, as in section 3.1, the equations that capture identities derived from the matrices are denoted by using an ‘i’ next to the equation number.

3.2.1 Firms

\[ Y = C + I \]  \hspace{1cm} (49)

\[ TP = Y - wL_{-1} - i_C LC_{-1} - i_G LG_{-1} - DES_C - DES_G \]  \hspace{1cm} (50i)

\[ RP = s_f TP \]  \hspace{1cm} (51)

\[ DP = TP - RP \]  \hspace{1cm} (52i)

\[ K = K_C + K_G \]  \hspace{1cm} (53)

\[ DES_C = \tau_4 K_{C-1} \]  \hspace{1cm} (54)

\[ DES_G = \tau_4 K_{G-1} \]  \hspace{1cm} (55)

\[ LF = LC + LG \]  \hspace{1cm} (56)

\[ r = RP/K \]  \hspace{1cm} (57)

\[ eq = E/K \]  \hspace{1cm} (58)

\[ lev = LF/K \]  \hspace{1cm} (59)

\[ Y^* = vK \]  \hspace{1cm} (60)

\[ u = Y/Y^* \]  \hspace{1cm} (61)

\[ L = L_1 + L_2 + L_3 + L_4 \]  \hspace{1cm} (62)

\[ re = L/LFO \]  \hspace{1cm} (63)
\[ I_C^{DES} = f\left( f_0, r_{-1}, u_{-1}, lev_{-1}, eq_{-1}, I_C \right) K_{C-1} + DES_C \] (64)

\[ I_G^{DES} = q\left( q_0, r_{-1}, u_{-1}, lev_{-1}, eq_{-1}, I_G, dep_{M-1}, dep_{E-1}, deg_{-1} \right) K_{G-1} + DES_G \] (65)

\[ \Delta K_C = I_C - DES_C \] (66)

\[ \Delta K_G = I_G - DES_G \] (67)

\[ \delta_C = \Delta K_C / K_{C-1} \] (68)

\[ \delta_G = \Delta K_G / K_{G-1} \] (69)

\[ e = e_{-1} + \frac{t_I}{p_e} \] (70)

\[ \beta = \frac{I_G}{I_C + I_G} \] (71)

\[ NLC^{DES} = I_C^{DES} - (1 - \beta)R - (1 - \beta)p_e \Delta e + repLC_{-1} - DES_C \] (72)

\[ NLG^{DES} = I_G^{DES} - \beta R - \beta p_e \Delta e + repLG_{-1} - DES_G \] (73)

\[ I_C^D = R + p_e \Delta e + \Delta LC + \Delta LG + DES_C + DES_G - I_G^D \] (74i)

\[ I_G^D = \beta R + \beta p_e \Delta e + \Delta LG + DES_G \] (75)

\[ I_C = \hat{\eta}_C^D \] (76)

\[ I_G = \hat{\eta}_G^D \] (77)

\[ \gamma = \begin{cases} 1, & \text{if } x_f^* \geq x_f^D \\ \frac{x_f^*}{x_f^D}, & \text{if } x_f^* < x_f^D \end{cases} \] (78)

\[ I = I_C + I_G \] (79)

Eq. (49) shows that the output of the economy is equal to the number of consumption goods \((C)\) plus the number of investment goods \((I)\). The total profits of firms \((TP)\) are given by equation (50i); \(w\) is the wage rate, \(L\) is the number of employed workers, \(i_c\) is the interest rate on conventional loans, \(i_G\) is the interest rate on green loans, \(LC\) is the amount of conventional loans, \(LG\) is the amount of green loans, \(DES_C\) is the destruction of conventional capital and \(DES_G\) is the destruction of green capital. Firms’ retained profits \((RP)\) are a proportion \((\gamma)\) of their total profits (Eq. 51). The distributed profits of firms \((DP)\) are determined as a residual (Eq. 52i). Eq. (53) shows that total capital \((K)\) is equal to conventional capital \((K_C)\) plus green
capital \( (K_G) \). Eqs. (54) and (55) give the destruction of conventional and green capital due to natural disasters. The total loans of firms \( (LF) \) are equal to conventional loans plus green loans (Eq. 56). Eq. (57) gives the rate of retained profits \( (r) \), Eq. (58) defines the value of equity \( (E) \) as a ratio of the capital stock \( (eq) \) and Eq. (59) gives the leverage ratio of firms \( (lev) \). The full-capacity output \( (Y^*) \) is a function of the capital stock (Eq. 60); \( v \) is the full-capacity output-to-capital ratio. The rate of capacity utilisation \( (u) \) is given in Eq. (61). The total number of employed workers \( (L) \) is equal to the sum of workers \( (L_1 + L_2 + L_3 + L_4) \) that are used as a fund in the economic processes conducted by firms (Eq. 62). The rate of employment \( (re) \) is given by equation (63); \( LFO \) is the labour force.

Eqs. (64) and (65) show that firms make two types of investment: (a) gross conventional investment \( (I^{DES}_C) \) that does not affect resource, recycling or pollution efficiency; (b) gross green investment \( (I^{DES}_D) \) that increases efficiency, as described in equations (15)-(20). The two types of investment have some common determinants: the rate of profit, the rate of capacity utilisation, the equity-to-capital ratio and the leverage ratio. Conventional investment is also affected by the term \( f_0 \) (which reflects the ‘animal spirits’ of the entrepreneurs) and the interest rate on conventional loans. Green investment is also affected by \( q_0 \) (which reflects environmental policies or environmental ethics), the interest rate on green loans and the depletion and the degradation ratios. The rationale behind the inclusion of the depletion and the degradation ratios is that, as the natural resources reduce or degrade, firms are induced to innovate in order to increase efficiency. Note that although we have not included the price of energy in the simplified model presented here, the positive impact of the energy depletion ratio on green investment is an implicit way to capture the positive link between the price of non-renewable resource and green investment. It is important to point out that our model incorporates explicitly the macroeconomic rebound effect (see, e.g. Barker et al, 2009 and Stern, 2011): whenever efficiency increases as a result of green investment, the resulting rise in total output produced partially offsets the initial beneficial effects on the ecosystem.

The change and the growth rate of the conventional and the green capital stock are given by Eqs. (66)-(69). A proportion \( (\nu) \) of firms’ investment expenditures are financed by issuing equities.

\[ \text{(66)-(69)} \]

\[ \nu \]
(Eq. 70); \( e \) is the number of firms' equities and \( p_e \) is their price. Eq. (71) defines the ratio (\( \beta \)) of green investment to total investment. For simplicity, it is assumed that this ratio determines the proportion of retained profits and equities used for financing green investment. Due to the existence of credit rationing, only a proportion of new loans that are demanded by firms are provided by banks. Eq. (72) gives the desired new conventional loans (\( NL^{DES} \)) and Eq. (73) gives the desired new green loans (\( NLG^{DES} \)). The investment goods that are demanded, after credit rationing has been imposed are shown in Eqs. (74i) and (75). \( I^D_C \) is the demanded conventional investment and \( I^D_G \) is the demanded green investment. However, due to supply-side constraints described in section 3.1.5, it is likely that not all demanded investment goods will be produced. In particular, this happens when \( x^*_f < x^D_f \). Eqs. (76)-(78) reflect this fact. The total produced investment goods are given by Eq. (79).

### 3.2.2 Households

\[
Y_H = wL_{-1} + DP + BP + i_D D_{-1} \tag{80}
\]

\[
C_D = c_1 Y_{H-1} + c_2 FV_{H-1} \tag{81}
\]

\[
C = \gamma C_D \tag{82}
\]

\[
\gamma = \begin{cases} 
1, & \text{if } x^*_f \geq x^D_f \\
\frac{x^*_f}{x^D_f}, & \text{if } x^*_f < x^D_f 
\end{cases} \tag{83}
\]

\[
\Delta FV_H = Y_H - C + e_{-1} \Delta p_e \tag{84}
\]

\[
\frac{p_e e}{FV_{H-1}} = \lambda_{10} + \lambda_{11} r_{e-1} + \lambda_{12} i_D + \lambda_{13} \frac{Y_{H-1}}{FV_{H-1}} \tag{85}
\]

\[
\frac{D}{FV_{H-1}} = \lambda_{20} + \lambda_{21} r_{e-1} + \lambda_{22} i_D + \lambda_{23} \frac{Y_{H-1}}{FV_{H-1}} \tag{86n}
\]

\[
D = FV_H - e p_e \tag{86}
\]

\[
r_e = \frac{RP + e_{-1} \Delta p_e}{p_{e-1} e_{-1}} \tag{87}
\]

\[
\Delta DC = C \tag{88}
\]
Eq. (80) gives the disposable income of households \( (Y_H) \); \( BP \) denotes the profits of banks, \( i_D \) is the interest rate on deposits and \( D \) is the amount of deposits. The number of goods that households wish to consume are given by Eq. (81). Consumption depends on lagged income (which is a proxy for the expected one) and lagged financial wealth \( (FV_H) \); \( c_1 \) is the propensity to consume out of income and \( c_2 \) is the propensity to consume out of financial wealth. It is important to point out that a rise in the price of equities can boost consumption expenditures and, hence, the output of the economy, with adverse effects on the ecosystem. Eqs. (82) and (83) follow the same rationale with Eqs. (76)-(78): when \( x_f^* < x_f^P \), only a proportion of the demanded consumption goods is produced.

Households allocate their expected financial wealth between deposits and firms’ equities. In the portfolio choice of households, Godley’s (1999) imperfect asset substitutability framework is adopted (Eqs. 85-86).\(^{16}\) Note that Eq. (86n) is necessary to be replaced with Eq. (86), with deposits acting as a buffer. Eq. (87) shows the rate of return on firms’ equity \( (r_e) \). Eq. (88) indicates the increase in durable consumption goods.

### 3.2.3 Banks
\[
CR_C = f\left(c_{0, lev-1, eq-1}\right) \quad (89)
\]
\[
CR_G = f\left(g_{0, lev-1, eq-1}\right) \quad (90)
\]
\[
NLC = (1 - CR_C)NLC^{DES} \quad (91)
\]
\[
NLG = (1 - CR_G)NLG^{DES} \quad (92)
\]
\[
\Delta LC = NLC - repLC_{-1} \quad (93)
\]
\[
\Delta LG = NLG - repLG_{-1} \quad (94)
\]
\[
BP = i_C LC_{-1} + i_G LG_{-1} - i_D D_{-1} \quad (95i)
\]
\[
i_C = spr_1 + i_D \quad (96)
\]
\[
i_G = spr_2 + i_D \quad (97)
\]
\[
D = LC + LG \quad (98\text{-red})
\]

\(^{16}\) The parameters in the portfolio choice equations satisfy the horizontal, vertical and symmetry adding-up constraints.
We define indicators of the degree of credit rationing for both conventional and green loans \((CR_c\text{ and } CR_g)\). These indicators lie between zero and one. The higher they are the higher the degree of credit rationing.\(^\text{17}\) As shown in Eqs. (89) and (90), these indicators are positively affected by the leverage ratio and negatively affected by the equity-to-capital ratio. In the case of green loans, \(g_0\) reflects the environmental practices of banks. The actual new loans and the actual change in loans (after credit rationing has been imposed) are given by Eqs. (91)-(94). The profits of banks are equal to the interest on both conventional and green loans minus the interest on deposits (Eq. 95). The interest rate on loans is equal to the interest rate on deposits plus a spread (Eqs. 96 and 97); \(spr_1\) and \(spr_2\) denote the spreads for conventional and green loans, respectively. Equation (98-red) is the redundant equation of the macroeconomic system described in Table 3 and Table 4: it is logically implied by all the other equations of this system.

4. Stock-flow-fund interactions between the ecosystem, the macroeconomy and the financial system

The ESFF framework presented in the previous sections provides a platform for the analysis of the complex interactions between the financial system, the macroeconomy and the ecosystem. Figure 1 illustrates the most important channels through which these interactions take place.

**Figure 1**: Stock-flow-fund interactions between the ecosystem, the macroeconomy and the financial system

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\(^{17}\) For similar formulations see Dafermos (2012).
The four channels through which the ecosystem interacts with the macroeconomy are the following:

**Depletion channel:** Macroeconomic activity leads to the extraction of non-renewable resources placing upward pressures on the energy and matter depletion ratios. This is captured by Eqs. (1), (2), (4), (5) and (6). This channel becomes less strong when resource and recycling efficiency increases as a result of green investment (see Eqs. 15-18).

**Degradation channel:** The harmful matter that is emitted due to macroeconomic activity (e.g. greenhouse gas emissions) tends to increase the degradation of the ecosystem services leading to climate change and natural disasters. In our model, this is captured by the upward pressures that macroeconomic activity places on the degradation ratio (see Eqs. 3 and 9). This channel is attenuated when pollution efficiency improves due to green investment (see Eqs. 19-20).

**Natural resources constraint channel:** As explained in detail above, the ecosystem provides the inflows of useful energy and matter that are necessary for the economic processes. These resources tend to reduce due to the depletion channel and they tend to increase as a result of discoveries of new stocks (see Eqs. 39i-40i). When useful energy and matter are not sufficient, the demand for investment and consumption goods might be satisfied only partially. This is captured by Eqs. (76)-(78) and Eqs. (82)-(83). It is also important to point out that, according to Eq. (65), a rise in the depletion ratios tends to increase green investment, weakening thereby the depletion channel.

**Damage channel:** The degradation of the ecosystem can affect the services of funds in the economic processes (labour and capital). Degradation can reduce labour productivity (see Eq. 22). This implies that at some point in time the labour force might not be sufficient to produce the final output demanded by households and firms. Moreover, degradation can result in natural disasters that destruct capital stock (see Eqs. 47 and 48). This might impose an additional supply-side constraint and it might reduce the profitability of firms (see Eq. 50i), with adverse effects on economic activity. Crucially, Eq. (45) shows that the higher the depletion the lower the absorption capacity of the ecosystem. This is a source of a destabilising dynamic in the degradation ratio.
The three channels through which the financial system interacts with the macroeconomy are the following:

**Financing of efficiency channel:** The financial system finances green investment via loans and equities (see Eqs. 74 and 75). In so doing, it contributes to the improvement of the efficiency indicators in the economic processes, with indirect favourable effects on the depletion and the degradation channel. In our model, the role of bank decisions is of particular importance: by reducing credit rationing (see Eqs. 89-90) and the interest rate on green loans they can contribute to ecological sustainability.

**Growth channel:** The financial system has both positive and negative effects on economic activity. The positive effects include the provision of finance that increases investment and the role that stock price increases can play in boosting investment and wealth-related consumption (see Eqs. 64-65 and Eq. 81). However, an increase in the leverage of firms and a reduction in stock prices can reduce the desired investment of firms and can increase credit rationing, reducing economic growth.

**Financial stability channel:** The stability of the financial system is affected by macroeconomic activity. However, the links are not clear-cut. High economic growth and low unemployment rates are conducive to the expansion of the financial system, which might be associated with higher financial fragility (reflected, for example, in higher leverage ratios). Low economic activity and high unemployment create debt repayment difficulties that effect the stability of the financial system. The model presented in section 3 does not include default and, therefore, does not capture explicitly the financial problems that may emerge from the real economy. However, simple extensions can incorporate such problems.

The channels described in Figure 1 can give rise to various complex and interesting dynamics. The ESFF modelling framework provides the adequate platform for the analysis of these dynamics, illuminating thereby the conditions under which financial stability, macroeconomic stability and ecological sustainability might be possible. Remarkably, the ESFF framework can be the basis for both small-scale and large-scale models. Small-scale models can focus on the examination of certain stock-flow-fund interactions, adopting simplifying assumptions for the issues that fall outside the purposes of the analysis. Large-scale models can extend the model
presented in section 3 by including, for instance, a central bank and a government, with the aim to assess the role of various environmental policies.

5. Concluding remarks

We put forward an ESFF modelling framework that integrates the post-Keynesian SFC modelling approach and the physical input-output accounting framework. The ESFF framework provides a coherent analysis of the monetary and physical stocks and flows using the accounting principles and the laws of thermodynamics. Moreover, it pays particular attention to the distinction between stock-flow resources and fund-service resources, it combines the post-Keynesian emphasis on the role of aggregate demand with ecosystem supply-side contraints and incorporates the non-neutral role of finance in the analysis of macroeconomic and ecological issues.

We used a benchmark model in order to illustrate how an ESFF model can be constructed. We then presented how the ESFF framework can be used in order to analyse the various stock-flow-fund channels through which the ecosystem, the macroeconomy and the financial system are interconnected. We argued that the high flexibility of the ESFF framework allows it to be used for various types of academic and policy analyses in the field of ecological macroeconomics.
References


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