Debt cycles, instability and fiscal rules:
A Godley-Minsky model

Yannis Dafermos*

Abstract: Wynne Godley and Hyman Minsky were two macroeconomists that ‘saw the crisis coming’. This paper develops a simple macrodynamic model that synthesises some key perspectives of their analytical frameworks. The model incorporates Godley’s ‘financial balances approach’ and postulates that the private expenditures are driven by a stock-flow norm that changes endogenously via a Minsky mechanism. The model is employed to explore the sources of debt cycles and instability. It is also used to study the (de)stabilising effects of a Maastricht-type fiscal rule that focuses on the stabilisation of government debt and a Godley-Minsky fiscal rule that links government expenditures with private indebtedness.

Keywords: Godley, Minsky, debt cycles, instability, fiscal rules

JEL Classification: E10; E20; E32; E62

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1. Introduction

Wynne Godley and Hyman Minsky were two macroeconomists that ‘saw the crisis coming’. In 1999 Godley published his well-known article on the ‘seven unsustainable processes’ in the US economy (Godley, 1999). In this article he argued that the rising private indebtedness in US was unsustainable and, therefore, private expenditures could not be considered as a source of steady growth in the medium term. He also pointed to the unsustainability of the rising US net foreign indebtedness. Using a stock-flow consistent analytical framework, Godley illustrated that without a change in fiscal policy stance or an important rise in net exports, the US economy was doomed to witness a severe recession and a sharp rise in unemployment. These warnings were repeated in his publications as a head of the Levy Economics Institute’s macro-modelling team (see, e.g. Godley, 2003, 2005; Godley et al., 2005). The 2007-9 crisis verified Godley’s fears: the US economy contracted rapidly and the unemployment rate increased substantially.2

Minsky (1975, 1982, 2008) developed a theory that explains how indebtedness can increase in periods of tranquillity as a result of endogenous forces that reduce the desired margins of safety of economic units. This gradual reduction in the desired margins of safety was considered by Minsky as the reason behind the increasing financial fragility that accompanies economic expansion and periods of stability. According to his ‘financial instability hypothesis’, the increasing fragility makes the macro systems more prone to shocks that reduce the ability of borrowers to repay their debt. These shocks can lead to severe economic recessions. The processes

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2 According to Godley et al. (2007) and Zezza (2009), the slowdown of economic growth in 2001-2 was a first sign of the unsustainable processes in the US economy. However, a severe recession was then prevented due to accommodative fiscal and monetary policies.
described in Minsky’s analysis are broadly in line with the pre-crisis developments in the US and other advanced economies that ultimately led to the Great Recession.³

The emphasis that Godley and Minsky placed on financial relationships as sources of cycles and instability enabled them to provide some very important insights into the dynamics of modern macroeconomies. However, they did so from quite different angles. Godley concentrated more on the macroeconomic relationships between the private, the government and the foreign sector and postulated that in the medium to long run the fluctuations in financial balances and growth are driven by some exogenous stock-flow norms. Minsky, on the other hand, focused more on the relationships within the private sector (primarily on the financial relationships between firms and banks) and explained the macroeconomic fluctuations by considering endogenous changes in norms and valuations of risk.

Although it is widely held that Godley’s and Minsky’s perspectives are both important for the explanation of macroeconomic dynamics, there is still a lack of a framework that synthesises them. The main objective of this paper is to make such a synthesis in a simple macrodynami model. The model concentrates on certain aspects of Godley’s and Minsky’s approaches that are deemed more important for the explanation of debt cycles and instability in a simplified skeleton that describes the dynamics of a national macroeconomy.

The key features of the model are the following. First, as in Godley’s projection analyses, the economy consists of three sectors: the private sector, the government sector and the foreign sector. This allows us to explicitly consider Godley’s ‘financial balances approach’ that explains the interlinkages between these sectors and the resulting effects on debt accumulation and growth. Note that Godley’s ‘financial balances approach’ is broadly in line with Kalecki’s ‘profit equation’ that was used by Minsky. Second, the private expenditures in the model are driven by a stock-flow norm (the target net debt-to-income ratio). Following Minsky, it is assumed that this norm varies endogenously as a result of changes in the expectations and conventions of borrowers and lenders during the economic cycle.

³ For some recent formulations of specific aspects of Minsky’s theoretical framework see Ryoo (2010) and Keen (2013).
Both Godley and Minsky emphasised the role of fiscal policy as a stabilising mechanism for the inherent unstable macro systems. An additional purpose of this paper is, therefore, to examine the implications of the constructed Godley-Minsky model for the conduct of fiscal policy. We do so by comparing the (de)stabilising effects of two different fiscal rules: a Maastricht-type fiscal rule that concentrates on the stabilisation of government debt and a Godley-Minsky fiscal rule that links government expenditures with private indebtedness.

The paper proceeds as follows. Section 2 sets out the structure of the model. Section 3 explores the interaction between private expenditures and net private indebtedness when the target net private debt-to-income ratio is exogenous and fiscal policy is neutral. Section 4 endogenises the target net private debt-to-income ratio and explores the implications for instability. Section 5 introduces the fiscal rules and examines their (de)stabilising effects. Section 6 summarises and concludes.

2. Structure of the model

Table 1 portrays the transactions matrix of our three-sector economy. National accounting implies:

\[ Y = P + G + X - M \]  
\[ Y_P = Y - T - rD_P \]  
\[ \dot{D}_P = -B_P = P - Y_P \]  
\[ \dot{D}_G = -B_G = G - T + rD_G \]  
\[ \dot{D}_F = -B_F = X - M - r(D_G + D_P) \]  
\[ \dot{D}_P + \dot{D}_G + \dot{D}_F = -(B_P + B_G + B_F) = 0 \]

where \( Y \) is the output of the economy, \( P \) stands for the private expenditures (consumption plus investment), \( G \) denotes government expenditures, \( X \) stands for exports, \( M \) denotes imports, \( Y_P \) is the disposable income of the private sector, \( D_P \) is the net private debt, \( B_P \) is the balance of the private sector, \( B_G \) is the balance of the
government sector, $T$ denotes taxes, $r$ is the interest rate, $D_G$ is the net government debt, $B_F$ is the balance of the foreign sector and $D_F$ is the net foreign debt. Note that the net debt of each sector is equal to its liabilities minus its assets. A sector is a net debtor when its net debt is positive and a net creditor when its net debt is negative.

Table 1. Transactions matrix.

<table>
<thead>
<tr>
<th></th>
<th>Private sector</th>
<th>Government sector</th>
<th>Foreign sector</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government expenditures</td>
<td>+G</td>
<td>-G</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Taxes</td>
<td>-T</td>
<td>+T</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Exports</td>
<td>+X</td>
<td></td>
<td>-X</td>
<td>0</td>
</tr>
<tr>
<td>Imports</td>
<td>-M</td>
<td></td>
<td>+M</td>
<td>0</td>
</tr>
<tr>
<td>Private expenditures</td>
<td>+P</td>
<td></td>
<td>-P</td>
<td>0</td>
</tr>
<tr>
<td>Private sector's income</td>
<td>-Y_p</td>
<td>+Y_p</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Interest</td>
<td>-rD_p</td>
<td></td>
<td>-rD_G</td>
<td>+r(D_p + D_G)</td>
</tr>
<tr>
<td>Change in net debt</td>
<td>+D_p</td>
<td>+D_G</td>
<td>+D_F</td>
<td>0</td>
</tr>
<tr>
<td>Total</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

For simplicity, the following assumptions have been made: the interest rate is exogenously determined by the monetary authorities and is the same for both the private and the government net debt; the government expenditures refer only to the purchase of goods and services provided by the private sector; the price level is set equal to unity and there are no changes in asset prices and exchange rates that affect the value of assets and liabilities (and, thus, the value of net debt).

Particular attention should be paid to identity (6). This identity reflects Godley’s ‘financial balances approach’.\(^4\) It states that the sum of the balances of the three sectors of the economy is equal to zero. This identity has been widely used by Godley himself and other economists to analyse the macroeconomic developments in various countries (see e.g. Godley, 1995, 1999, 2003, 2005; Godley et al., 2007; Zezza, 2009; Sawyer, 2011; Brecht et al., 2012; Wolf, 2012). The important implication of this identity is that the balance of one sector cannot improve without a deterioration in the balance of at least one of the other two sectors. Therefore, if, for example, the government sector desires to decrease its deficit to a specific level then the private sector and/or the foreign sector should be willing to accept a deterioration

of their balances in an accurately offsetting manner. Otherwise, the intended decline in deficit cannot be attained. Moreover, since the main components of the financial balances are also components of the aggregate demand any attempt of the sectors to improve their balances may lead to lower output if the other sectors do not desire to experience lower balances.

A distinguishing feature of the ‘financial balances approach’ is the consolidation of households, firms and banks into one single private sector. This implies that in our model the transactions between households, firms and banks are not taken into consideration. Moreover, the assets and liabilities of the private subsectors that are counterparts of the assets and liabilities of other private subsectors are netted out in the estimation of the net private debt. The net private debt refers, therefore, solely to the net liabilities of the private sector that are net assets of the government and the foreign sector.

This consolidation is a great simplification with various limitations (see Dos Santos and Macedo e Silva, 2010 and Martin, 2012 for a discussion). However, it has proved quite useful in Godley’s projections and other empirical analyses that focus on the interaction between private sector’s behaviour, fiscal policy and foreign balance. Moreover, it serves the purposes of our simple skeleton that intends to capture the dynamics of a national macroeconomy by using a high-level aggregation.

In the model we make the following definitions:

\[ p = \frac{P}{Y(1-\tau)} \]  
\[ g = \frac{G}{Y} \]  
\[ d_p = \frac{D_p}{Y(1-\tau)} \]  
\[ d_G = \frac{D_G}{Y} \]  

(7) (8) (9) (10)
where \( p \) is the propensity of the private sector to spend out of its income, \( g \) is the ratio of government expenditures-to-output, \( d_p \) is the net private debt-to-income ratio and \( d_{go} \) is the net government debt-to-output ratio.

The following behavioural assumptions are made:

\[
T = \tau \cdot Y \quad (11)
\]
\[
M = m \cdot Y \quad (12)
\]
\[
\dot{X} = g_x X \quad (13)
\]
\[
\dot{p} = \lambda \left( d_p^T - d_p \right) \quad (14)
\]
\[
\dot{d}_p^T = \theta_1 (g_Y - g_{Y0}) + \theta_2 (d_p^B - d_p^T) \quad (15)
\]

Eqs. (11) and (12) imply that the taxes and the imports are proportional to the income of the economy \((\tau, m > 0)\). Eq. (13) shows that, for simplicity, the exports grow at an exogenously given rate, \( g_x \). This rate relies on factors such as the economy’s structural competiveness and the income of the foreign sector, which are taken as given.

Eq. (14) draws on Godley’s hypothesis about the behaviour of the private sector. Godley argued that the private sector targets in the long run a specific stock of net financial assets as a proportion of its disposable income (a stock-flow norm). He also postulated a formula which states that the balance of the private sector adjusts in order for this desired stock to be attained (see Godley and Cripps, 1983; Godley, 1999; Godley and Lavoie, 2007).\(^6\)

In our setup, \( d_p^T \) expresses the targeted net private debt-to-income ratio. Importantly, it is considered that this target in not only set by the private sector itself. It is also set by the government and, most importantly, by the foreign sector who are potentially lenders of the private sector. For instance, it may capture the willingness of foreign investors to lend to the private sector of a national economy (households, firms or

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\(^5\) For simplicity, the income before the interest payments is used.

\(^6\) See also Martin (2012) and Shaikh (2012).
banks). Therefore, this target is affected by the decisions of both borrowers and lenders.\(^7\)

Formula (14) implies that the private sector’s propensity to spend out of its disposable income increases (decreases) when the actual net debt ratio is lower (higher) than the targeted one (i.e. \(\lambda > 0\)). Notice that a change in the propensity to spend may express the decisions of both the private sector and its lenders. Although a change in the propensity to spend is the primary means through which the private sector can affect its net indebtedness, it may not have the desired outcomes. As will become clear below, the decision for spending affects the output of the economy and therefore has feedback effects on the net private debt-to-income ratio.

Interestingly enough, formula (14) shares some similarities with the recent macroeconomic analysis of Koo (2013) about what he calls a ‘balance sheet recession’. In this analysis Koo makes a distinction between periods in which the private sector maximises profits (‘Yang phases’) and periods in which the private sector minimises its debt (‘Yin phases’). In our model, the reduction in the propensity of the private sector to spend when \(d_p^T < d_p\) resembles a ‘Yin phase’ à la Koo.

Moreover, Eq. (14) can capture changes in private expenditures caused by capital inflows and capital outflows. For example, the inequality \(d_p^T > d_p\) may reflect periods in which the net debt of the private sector is considered by foreign lenders as sufficiently small. In such periods the existence of a low perceived lender’s risk induces higher capital inflows that lead to higher private expenditures relative to income. On the other hand, the inequality \(d_p^T < d_p\) may capture periods of capital outflows in which the lender’s risk is perceived to be high.

Eq. (15) draws on Minsky. Minsky (2008, pp. 193, 209) argues that during periods of expansion, when the outstanding debts are serviced without significant problems, the desired margins of safety of borrowers and lenders become lower. This happens

\(^7\) Note that the private sector can be either a net debtor (when the net private debt is positive) or a net creditor (when the net private debt is negative).
because the recent good performance of the economy and the favourable credit history induce economic units to accept financial structures that were previously assessed as risky.\textsuperscript{8} The opposite holds in periods in which the economic performance and credit history are not favourable. Although Minsky’s arguments basically refer to the behaviour of firms and banks, they can be applied to any borrower-lender relationship and therefore, to the financial relationships between the private sector of a national economy and its lenders/borrowers (the government and the foreign sector).

Based on this Minskyan perspective, Eq. (15) states that when economic growth ($g_Y$) is higher (lower) than a benchmark growth rate ($g_{Y0}$), the target net debt-to-income ratio of the private sector increases (decreases); note that $\theta_1 > 0$.\textsuperscript{9} By endogenising the target net private debt-to-income ratio, Eq. (15) can be viewed as a Minskyan extension of Godley’s exogenous stock-flow norm for the private sector. In section 4 it will be shown that the endogeneity of the stock-flow norm can increase the amplitude of debt cycles and can be conducive to higher instability.

However, economic growth is not the only driver of the target net debt ratio. According to Eq. (15), this target is also partially attracted by a benchmark net debt ratio, $d_P^B$, which is constant and depends on deep institutional factors (e.g. the degree of financial development, the trade institutions, the political relationships of the country under investigation with other countries etc.); note that $\theta_2 > 0$. When $d_P^B > 0$, the private sector has a net debtor benchmark position; when $d_P^B < 0$, it has a net creditor benchmark position.

The dynamic behaviour of $g$ is determined by fiscal rules. Fiscal rules have been quite fashionable over the last decades and have been more widely adopted since the outbreak of the crisis (see Schaechter et al., 2012). Typically, these rules impose constraints on government debt and deficit.\textsuperscript{10} We first consider a simple Maastricht-

\textsuperscript{8} See also Kregel (1997) and Tymoigne (2009).
\textsuperscript{9} For a similar formulation that focuses on the endogeneity of the desired margins of safety of firms and banks see Nikolaidi (2014).
\textsuperscript{10} For a recent review of the use of fiscal rules in the conventional macroeconomic literature see Chortareas (2013).
type rule which states that government expenditures (relative to output) decline when the net government debt-to-output ratio is higher than a specific target \((d_G^T)\). Formally, this rule is written as follows:

\[
g = \mu(d_G^T - d_G)
\]

(16a)

where \(\mu > 0\).

We then consider an alternative fiscal rule that departs from the conventional approach since it places no limits on any specific fiscal aggregate. On the contrary, its rationale is that fiscal policy should stabilise the macroeconomy by increasing (decreasing) government expenditures when the private sector attempts to reduce (increase) its indebtedness producing contractionary (expansionary) effects. Algebraically:

\[
g = -\kappa(d_P^T - d_P)
\]

(16b)

where \(\kappa > 0\).

Eq. (16b) is consistent with the perceptions of both Godley and Minsky who emphasised that the government should intervene to offset fluctuations in economic activity that stem from the inherently unstable behaviour of the private sector.\(^{11}\) We thus call Eq. (16b) a Godley-Minsky fiscal rule.

Combining Eqs. (1), (7), (8), (11) and (12) we get:

\[
Y = pY(1 - \tau) + gY + X - mY
\]

(17)

Solving the above equation for \(Y\) yields:

\(^{11}\) Eq. (16b) is also in line with Koo’s (2013) suggestions that during ‘Yin phases’ fiscal expansion is necessary in order to avoid deep recessions, with the opposing holding during ‘Yang phases’.
The denominator in the above equation must be positive to ensure goods market stability (i.e. \(1+m-p(1-\tau)-g > 0\)).

Differentiating Eq. (18) with respect to time, and dividing through by \(Y\) gives the growth rate of the economy:

\[
\frac{\dot{Y}}{Y} = g_Y = g_X + \frac{p(1-\tau)+g}{1+m-p(1-\tau)-g}
\]

(19)

We also have that:

\[
\dot{d}_p = \frac{\dot{D}_p}{Y(1-\tau)} - d_p \frac{\dot{Y}}{Y} = p - 1 + (r-g_Y)d_p
\]

(20)

And:

\[
\dot{d}_G = \frac{\dot{D}_G}{Y} - d_G \frac{\dot{Y}}{Y} = g - \tau + (r-g_Y)d_G
\]

(21)

Eqs. (20) and (21), in conjunction with (19), show that when the private and the government sector decide to spend less (relative to income) the impact on their net debt-to-income ratios depends on whether they are net debtors or net creditors. When the net debt is positive, there are two counteracting effects. On the one hand, the decline in expenditures (i.e. in \(p\) and \(g\)) tends to reduce the net debt-to-income ratios. We call this the ‘spending effect’. On the other hand, such a decline reduces \(g_Y\), which in turn places upward pressures on the positive net debt-to-income ratios by reducing their denominator. We call this the ‘growth effect’. However, when the net debt is negative, these two effects are mutually reinforcing. The reason is that a lower \(g_Y\) reduces the denominator in the negative net debt-to-income ratios making them more negative.
The balances of the three sectors as a proportion of the output are as follows:

\[ b_P = \frac{B_P}{Y} = (1 - p - rd_P)(1 - \tau) \]  
(22)

\[ b_G = \frac{B_G}{Y} = \tau - g - rd_G \]  
(23)

\[ b_F = \frac{B_F}{Y} = m - \frac{X}{Y} + r[d_P(1 - \tau) + d_G] \]  
(24)

Note that \( b_P + b_G + b_F = 0 \) \(^{12}\)

Eqs. (14), (15), (16), (20) and (21) constitute a 5D dynamic system, which is reproduced below for convenience:

\[ \dot{d}_p = p - 1 + (r - g_Y)d_P \]  
(20)

\[ \dot{p} = \lambda (d_P^T - d_P) \]  
(14)

\[ \dot{d}_G = \theta_1 (g_Y - g_{Y0}) + \theta_2 (d_P^T - d_P^T) \]  
(15)

\[ \dot{d}_G = g - \tau + (r - g_Y)d_G \]  
(21)

\[ \dot{g} = \mu (d_G^T - d_G) \]  
(16a)

\[ \dot{g} = -\kappa (d_P^T - d_P) \]  
(16b)

The steady-state values for the endogenous variables of the system (denoted by the subscript 0) are as follows:

\[ d_{p0} = d_{p0}^T = d_P^B \]  

\[ d_{G0} = d_G^T \]  

\[ p_0 = 1 + (g_X - r)d_P^B \]  

\[ g_0 = \tau + (g_X - r)d_G^T \]  

Note that \( d_P^B \) and \( d_G^T \) are parameters. At the steady state it also holds that:

\(^{12}\) This can be easily shown by substituting in Eq. (24) the ratio \( X/Y \) from Eq. (18).
\[ b_{P0} = -g_X d_P^R (1 - \tau) \]
\[ b_{G0} = -g_X d_G^T \]
\[ b_{F0} = g_X \left[ d_P^R (1 - \tau) + d_G^T \right] \]

Prior to proceeding to the analysis of the dynamic macro system and its subsystems it is useful first to briefly examine the law of motion of private and government net indebtedness when economic growth is exogenous. This can be done by assuming that \( p \), \( g \) and \( d_G^T \) are at their steady-state values.

Under these conditions, from Eqs. (20) and (21) we get:

\[ \frac{\partial \dot{d}_P}{\partial d_P} = r - g_X \]
\[ \frac{\partial \dot{d}_G}{\partial d_G} = r - g_X \]

Therefore, the net private debt-to-income ratio and the net government debt-to-output ratio are stable when \( g_X > r \). This implies that when economic activity is exogenous, the stability of the net debt ratios relies on the export performance of the economy and the stance of monetary policy. The lower the interest rate set by monetary authorities and the higher the export growth of the national macroeconomy, the higher the likelihood that the debt ratios will stabilise.

In the dynamic analysis that follows it will be assumed that \( g_X > r \). This will allow us to confine attention to the destabilising forces that stem from the Godleyan and Minskyan mechanisms described above. The parameter values used in the simulation exercises are reported in Appendix A.\(^{13}\)

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\(^{13}\) The simulation exercises have been conducted using the Matlab software programme.
3. The 2D subsystem: interaction between net private indebtedness and private spending

This section analyses the 2D subsystem consisting of Eqs. (14) and (20). Our aim is to examine the dynamic interaction between net private indebtedness and private spending when the target net private debt-to-income ratio is exogenous and the fiscal policy is neutral. Hence, \( g \) and \( d_p^T \) are kept at their steady-state values (i.e. \( g = g_0 \) and \( d_p^T = d_p^0 \)); \( d_G \) is not necessary to be kept at its steady-state value since it does not feed back into the law of motion of \( d_p \) and \( p \).

The Jacobian matrix of the 2D subsystem evaluated at the steady state is:

\[
J_{2D} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}
\]

where:

\[
J_{11} = \frac{\partial \dot{d}_p}{\partial d_p} = r - g_X + \frac{d_p^R(1-\tau)\dot{\lambda}}{1 + m - p_0(1-\tau) - g_0}
\]

\[
J_{12} = \frac{\partial \dot{d}_p}{\partial p} = 1
\]

\[
J_{21} = \frac{\partial \dot{p}}{\partial d_p} = -\dot{\lambda}
\]

\[
J_{22} = \frac{\partial \dot{p}}{\partial p} = 0
\]

We have that:

\[
\text{Tr}(J_{2D}) = J_{11} + J_{22} = r - g_X + \frac{d_p^R(1-\tau)\dot{\lambda}}{1 + m - p_0(1-\tau) - g_0}
\]

\[
\text{Det}(J_{2D}) = J_{11}J_{22} - J_{21}J_{12} = \dot{\lambda} > 0
\]

It can be readily seen that since \( g_X > r \), the sign of \( \text{Tr}(J_{2D}) \) depends on the sign of \( d_p^R \). If \( d_p^R > 0 \), then \( \text{Tr}(J_{2D}) > 0 \) for sufficiently high values of \( \dot{\lambda} \). Therefore, when

\[14 \text{ Recall that } 1 + m - p_0(1-\tau) - g_0 > 0.\]
the private sector has a net debtor benchmark position, the more its propensity to spend responds to the divergence between the actual and the target net private indebtedness the higher, ceteris paribus, the likelihood that the system is unstable. The rationale behind this result is straightforward: when the net private debt ratio is higher (lower) than the target one, any attempt of the private sector to reduce (increase) its indebtedness by reducing (raising) the propensity to spend has an adverse (favourable) impact on economic growth. For sufficiently high values of $\lambda$, this ‘growth effect’ dominates the ‘spending effect’ leading to instability.

If $d_p^B < 0$, then $Tr(J_{2D}) < 0$ and the system is always stable: the stabilising impact of a change in the propensity to spend is reinforced by the associated ‘growth effect’ (see section 2). Consequently, when the private sector has a net creditor benchmark position, Eq. (14) does not produce destabilising forces.

Let us analyse in greater detail the case in which $d_p^B > 0$. By setting $Tr(J_{2D}) = 0$ we can find the critical value for $\lambda$ above which the system becomes unstable:

$$\lambda^* = \frac{(r-g_s)(1+m-p_0(1-\tau)-g_0)}{d_p^B (1-\tau)} > 0$$

Instability emerges when $\lambda > \lambda^*$. The system is stable when $\lambda < \lambda^*$. Interestingly, $\lambda^*$ increases as the interest rate becomes lower and the growth rate of exports becomes higher. This implies that adequate monetary and trade policy can, until some limit, prevent the destabilising forces that stem from the behaviour of the private sector.

Moreover, it can be shown that for a wide range of $\lambda$ values (higher than $\lambda^*$) the 2D subsystem exhibits unstable cycles (see Appendix A for a proof). Figure 1 presents the unstable cycles in our simulation analysis.$^{15}$ In the simulations it holds that $d_p^B = d_p^T = 0.25$. Assume that the economy is initially in phase I. Since $d_p < d_p^T$, the private sector increases its propensity to spend producing higher than steady-state growth. Simultaneously, net private indebtedness declines because the propensity to spend is not high enough. Phase I can be interpreted as a phase of recovery. As the

$^{15}$ In the simulations $\lambda^* = 0.008$.  

15
net private debt-to-income ratio declines, \( p \) continues to increase and eventually the economy enters phase II in which the propensity to spend is high enough to generate a rise in indebtedness. In this phase the economy continues to exhibit a high growth which, however, is accompanied by higher fragility. At some point, \( d_p \) becomes higher than \( d^* \). At that point the indebtedness of the private sector is conceived to be extremely high from the borrowers’ and/or lenders’ perspective; this causes a reduction in the propensity of the private sector to spend. The economy enters a period of stagnation (phase III) where low growth coexists with rising net indebtedness. This rising indebtedness reduces further the private sector’s propensity to spend. Indebtedness starts declining only when the propensity to spend is low enough to outweigh the adverse affects of low growth on the debt ratio. When this happens the economy enters a new phase (phase IV) where the economic growth remains low (since \( d_p \) is still higher than \( d^* \)). However, declining indebtedness sets the stage for the recovery that occurs when \( d_p \) falls short of \( d^* \). When this happens, a new cycle begins.

\textbf{Fig. 1.} Unstable cycles in the 2D subsystem, private sector in a net debtor benchmark position \((d_p^b > 0)\).

Since the cycles are unstable, economic fluctuations become gradually more severe: in every new cycle the deviation of the net private debt-to-income ratio from its target level becomes higher. Similarly, the propensity to spend deviates more from
its steady-state value. Therefore, the more the private sector and its lenders attempt to put net private indebtedness under control by adjusting private expenditures the more the private debt ratio destabilises. Arguably, this is a ‘paradox of debt’ result: the macroeconomic effects of the change in expenditures prevent the realisation of the desired indebtedness. \(^{16}\)

The instability generated as a result of the endogenous changes in \(p\) and \(d_p\) can be also shown in Fig. 2. Note that since \(g\) is constant, net government indebtedness is exclusively driven by the ‘growth effect’: when economic growth is high (low) enough the net government debt-to-output ratio declines (increases). Moreover, the government financial balance does not change significantly. Consequently, any deterioration or improvement in the financial balance of the private sector is almost entirely mirrored in the balance of the foreign sector.

---

\(^{16}\) For the ‘paradox of debt’ in the case of firms see Steindl (1952), Lavoie (1995), Hein (2007) and Ryoo (2013).
Fig. 2. Dynamic adjustments of the 2D system to a 5% increase in the net private debt-to-income ratio, private sector in a net debtor benchmark position ($d_p^B > 0$).
4. Endogenising the targeted net private indebtedness

We now allow the target net private debt-to-income ratio to change endogenously according to formula (15); \( g \) is still kept at its steady-state value. It will be shown that the introduction of Eq. (15) into an otherwise stable 2D subsystem can lead to instability for sufficiently high values of \( \theta_1 \).

The 3D subsystem of Eqs. (14), (15) and (20) has the following Jacobian matrix (evaluated at the steady state):

\[
J_{3D} = \begin{pmatrix}
J_{11} & J_{12} & J_{13} \\
J_{21} & J_{22} & J_{23} \\
J_{31} & J_{32} & J_{33}
\end{pmatrix}
\]

where:

\[
J_{11} = \frac{\partial \dot{d}_p}{\partial d_p} = r - g_X + \frac{d_p^B(1-\tau)\lambda}{1 + m - p_0(1-\tau) - g_0}
\]
\[
J_{12} = \frac{\partial \dot{p}}{\partial d_p} = 1
\]
\[
J_{21} = \frac{\partial \dot{p}}{\partial \dot{d}_p} = -\lambda
\]
\[
J_{22} = \frac{\partial \dot{p}}{\partial \dot{p}} = 0
\]
\[
J_{13} = \frac{\partial \dot{d}_p}{\partial \dot{d}_p} = -\frac{(1-\tau)d_p^B\lambda}{1 + m - p_0(1-\tau) - g_0}
\]
\[
J_{23} = \frac{\partial \dot{p}}{\partial \dot{d}_p} = \lambda
\]
\[
J_{31} = \frac{\partial \dot{d}_p}{\partial \dot{d}_p} = -\frac{(1-\tau)\lambda\theta_1}{1 + m - p_0(1-\tau) - g_0}
\]
\[
J_{32} = \frac{\partial \dot{d}_p}{\partial \dot{p}} = 0
\]
\[
J_{33} = \frac{\partial \dot{d}_p}{\partial \dot{d}_p} = \frac{\theta_1(1-\tau)\lambda}{1 + m - p_0(1-\tau) - g_0} - \theta_2
\]

The characteristic equation of the system is:

\[
\Gamma(d) = d^3 + a_1d^2 + a_2d + a_3
\]
We have that:

(i) \( a_1 = -(J_{11} + J_{22} + J_{33}) \) or
\[ a_1 = -\text{Tr}(J_{2D}) + \theta_2 - \frac{\theta_1(l - \tau)}{1 + m - p_0(l - \tau) - g_0} \quad \text{or} \quad a_1 = \Omega_1 + \Omega_2 \theta_1 \]

where \( \Omega_1 = -\text{Tr}(J_{2D}) + \theta_2 \) and \( \Omega_2 = -\frac{(l - \tau)}{1 + m - p_0(l - \tau) - g_0} < 0 \).

(ii) \( a_2 = \begin{vmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{vmatrix} \quad \text{or} \quad a_2 = \lambda + \text{Tr}(J_{2D}) \frac{\theta_1(l - \tau)}{1 + m - p_0(l - \tau) - g_0} - \theta_2 \frac{\theta_1(l - \tau)^2 x_p^2}{1 + m - p_0(l - \tau) - g_0} \quad \text{or} \quad a_2 = \Omega_3 + \Omega_4 \theta_1 \]

where \( \Omega_3 = \lambda - \text{Tr}(J_{2D}) \theta_2 \) and \( \Omega_4 = \frac{(r - x_p) \lambda (l - \tau)}{1 + m - p_0(l - \tau) - g_0} < 0 \).

(iii) \( a_3 = -\text{Det}(J_{3D}) = -J_{11} J_{22} J_{33} - J_{12} J_{21} J_{33} - J_{13} J_{12} J_{21} > 0 \)

(iv) \( b = a_1 a_2 - a_3 \quad \text{or} \quad b = (\Omega_1 + \Omega_2 \theta_1)(\Omega_3 + \Omega_4 \theta_1) - a_3 \)

According to the Routh-Hurwitz stability conditions, the 3D subsystem is stable if \( a_1, a_2, a_3, b > 0 \). Since we consider the case in which the 2D subsystem is stable, \( \text{Tr}(J_{2D}) < 0 \) and therefore \( \Omega_1, \Omega_3 > 0 \). We have that \( a_1 \) and \( a_2 \) are a decreasing function of \( \theta_1 \); \( a_3 \) is independent of \( \theta_1 \). By setting \( a_1 = 0 \) and solving for \( \theta_1 \), we obtain \( \theta_1^{\text{th}} = -\Omega_1 / \Omega_3 > 0 \). For \( \theta_1 > \theta_1^{\text{th}} \), \( a_1 \) becomes negative. By setting \( a_2 = 0 \) and solving for \( \theta_1 \), we obtain \( \theta_1^{\text{th}} = -\Omega_3 / \Omega_4 > 0 \). For \( \theta_1 > \theta_1^{\text{th}} \), \( a_2 \) becomes negative.

The expression for \( b \) can be written as:

\[ b = \Omega_2 \Omega_4 \theta_1^2 + (\Omega_1 \Omega_4 + \Omega_2 \Omega_3) \theta_1 + \Omega_4 \Omega_3 - a_3 \]

We obtain two roots:
\[ \theta_i^b, \theta_i^c = \frac{-(\Omega_4 + \Omega_2 \Omega_3) \pm \sqrt{((\Omega_4 + \Omega_2 \Omega_3)^2 - 4\Omega_2 \Omega_4 (\Omega_2 \Omega_3 - a_i)}}{2\Omega_2 \Omega_3} \]

It can be proved that \((\Omega_4 + \Omega_2 \Omega_3)^2 - 4\Omega_2 \Omega_4 (\Omega_2 \Omega_3 - a_i) > 0\). This implies that the two roots are real. We have that:

\[
b = \Omega_2 \Omega_3 [(a_i - \theta_i^b) (a_i - \theta_i^c)]
\]

According to Vieta’s formulas, \(\theta_i^b + \theta_i^c = -(\Omega_4 + \Omega_2 \Omega_3)/(\Omega_2 \Omega_4) > 0\) and \(\theta_i^b \theta_i^c = (\Omega_2 \Omega_3 - a_i)/(\Omega_2 \Omega_4) > 0\) (it can be proved that \(\Omega_2 \Omega_3 - a_i > 0\)). Therefore, \(\theta_i^b\) and \(\theta_i^c\) are both positive. Assuming that \(\theta_i^{bi} < \theta_i^{ci}\), it follows that \(b < 0\) if \(\theta_i^{bi} < \theta_i < \theta_i^{ci}\). Otherwise, \(b > 0\).

Overall, if \(\theta_i^{bi} < \min(\theta_i^{ci}, \theta_i^{bi}) < \theta_i^{ci}\), the 3D subsystem becomes unstable for \(\theta_i > \theta_i^{bi}\). Otherwise, the system becomes unstable for \(\theta_i > \min(\theta_i^{ci}, \theta_i^{bi})\). Thus, for a sufficiently high value of \(\theta_i\) an otherwise stable system becomes unstable: the more the target net private debt-to-income ratio responds to changes in economic growth the higher the likelihood of instability in the macroeconomy. This holds irrespective of the sign of \(d^B\).

Fig. 3 illustrates this in our simulations.\(^{17}\) The figure refers to a private sector that has a net creditor benchmark position (similar results hold when a net debtor benchmark position is considered). It can be seen that the stability properties of the 3D subsystem change as \(\theta_i\) increases: although the system is stable for low values of \(\theta_i\), it produces unstable cycles when \(\theta_i\) becomes higher than a specific threshold.\(^{18}\)

The underlying mechanism can be explained as follows. In periods of low growth, when net private indebtedness is high, the deterioration in borrowers’ and lenders’ expectations induces them to target a lower net debt ratio than the benchmark one

\(^{17}\) For similar figures that capture destabilising effects of specific equations in dynamic systems see Chiarella et al. (2012) and Nikolaidi (2014).

\(^{18}\) In the simulations the system becomes unstable for \(\theta_i > 0.342\).
\((d_p^B)\); recall that in the 2D subsystem \(d_p^T = d_p^B\). Therefore, the difference between the actual and the target ratio increases, producing a greater decline in the propensity to spend (and therefore in economic growth) compared to the 2D subsystem. Inversely, in periods of high growth, in which net private indebtedness is low, the favourable expectations due to the good performance of the economy make the perceived risk lower. This leads to a higher target net debt ratio than the benchmark one and, hence, to a more important rise in the propensity to spend. This results in higher economic growth. The greater fluctuations in both the propensity to spend and economic growth are reflected in the law of motion of the net debt ratio leading, overall, to unstable cycles.
Fig. 3. Dynamic adjustments of the 3D system to a 5% increase in the net private debt-to-income ratio for varying values of $\theta_1$, private sector in a net creditor benchmark position ($d^b_P < 0$).

(a) Net private debt-to-income ratio ($d_P$)

(b) Private sector’s propensity to spend ($p$)

(c) Target net private debt-to-income ratio ($d^T_P$)

(d) Net government debt-to-output ratio ($d_G$)

(e) Growth rate ($g_Y$)

(f) Government balance-to-output ratio ($b_G$)
5. The 5D system: introducing fiscal rules

We now turn to examine how the stability of the macro system changes when fiscal rules are introduced. Fig. 4 illustrates the dynamic adjustment of the system when fiscal authorities adopt a Maastricht-type fiscal rule (Eq. 16a). Fig. 5 shows the dynamic adjustments when a Godley-Minsky fiscal rule is implemented (Eq. 16b). In both simulation exercises the parameter values that refer to Eqs. (14), (15) and (20) are the same with those used in the simulations presented in Fig. 3; the same range of values for \( \theta_i \) has also been employed.\(^{19}\) This allows us to specify how the dynamic adjustments of the macro system are modified as a result of the introduction of fiscal rules.\(^{20}\)

Let us first focus attention on the Maastricht-type fiscal rule. Comparing the simulation results between Fig. 3 and Fig. 4, it can be easily observed that instability increases as a result of the implementation of this fiscal rule: for high values of \( \theta_i \) the cycles become much more intense. Intuitively, the following mechanisms are at play. Whenever economic growth is low (as a result of high net private indebtedness) there is a tendency for the net government debt-to-output ratio to increase. At some point during the period of low growth, the government debt ratio becomes higher than \( d'^T \). To guarantee fiscal discipline the government responds by reducing the expenditures-to-output ratio.\(^{21}\) This magnifies the contractionary effects that stem from the behaviour of the private sector: at a first place, economic growth is adversely affected by the decrease in \( g \); at a second place, this additional decline in growth enhances the deterioration in the expectations reducing further \( d'^T \); other things equal, the divergence between \( d_p \) and \( d'^T \) increases with destabilising effects on growth, private expenditures and net private debt. When the private sector has a net debtor benchmark position there is an additional channel through which the difference between \( d_p \) and \( d'^T \) increases: lower growth resulted from fiscal stance places upward pressures on \( d_p \).

\(^{19}\) The figures refer again to the case in which \( d_p^B < 0 \). This enables us to make comparisons with the simulations presented in Fig. 3.

\(^{20}\) It should be mentioned that when the Godley-Minsky fiscal rule is utilised, the whole macro system is indeed a 4D system since \( d'^T \) has no feedback effects on the rest of the system.

\(^{21}\) In the simulations \( d'^T = 0.5 \).
The inverse mechanisms are at work when economic growth is high. This implies that the Maastricht-type fiscal rule increases the amplitude of debt and economic cycles.\footnote{For the destabilising effects of Maastricht-type fiscal rules see also Charpe et al. (2011, ch. 9).}

Importantly, the induced instability refers not only to the private economy but also to the government sector. Fig. 4 illustrates that, as time passes, the Maastricht-type fiscal rule generates significant fluctuations in both the net government debt-to-output ratio and the government balance (as a proportion of output); these fluctuations are much more severe than those observed in Fig. 3. This result stems from the amplification of the economic cycles described above. Therefore, in an economy in which the private expenditures respond to changes in net private indebtedness the currently fashionable ‘debt brake’ rules do not only seem to destabilise the private sector but they may also be ineffective in ensuring fiscal prudence. Actually, a ‘paradox of debt’ result emerges: the more the fiscal authorities attempt to target a specific government debt ratio by changing the government expenditures the more this ratio destabilises.
Fig. 4. Dynamic adjustments of the 5D system to a 5% increase in the net private debt-to-income ratio for varying values of $\theta_1$, private sector in a net creditor benchmark position ($d_p^B < 0$), Maastricht-type fiscal rule.

- (a) Net private debt-to-income ratio ($d_p$)
- (b) Private sector’s propensity to spend ($p$)
- (c) Target net private debt-to-income ratio ($d_p^T$)
- (d) Net government debt-to-output ratio ($d_G$)
- (e) Growth rate ($g_Y$)
- (f) Government balance-to-output ratio ($b_G$)
On the other hand, the Godley-Minsky fiscal rule suggested here is capable of stabilising both the private economy and the government sector for high values of $\theta_1$. Fig. 5 indicates this. After some fluctuations in the initial periods (which are much less intense than the fluctuations in Fig. 4) all macro variables converge towards their steady-state values. Economically, this can be explained as follows. When economic growth is low due to high net private indebtedness, the implementation of the Godley-Minsky fiscal rule produces a rise in the government expenditures-to-output ratio. This has favourable effects on economic growth since it tends to reduce the divergence between $d_p$ and $d_p^T$ by placing upward pressures on $d_p^T$; as alluded to before the reduction of this divergence is conducive to stability. In high growth phases the government expenditures-to-output ratio falls, slowing down the economic growth that is caused by the behaviour of the private sector. This again tends to reduce the difference between $d_p$ and $d_p^T$ via the impact on $d_p^T$. Consequently, fiscal policy narrows down the amplitude of the cycles by suppressing the destabilising forces that stem from the endogenous changes in the desired margins of safety. This is also beneficial for the government sector itself. Since after some periods the fluctuations in economic growth reduce, the same happens in the government balance (as a proportion of output) and the net government debt ratio. Therefore, although at a first place there might be some adverse developments in the fiscal performance, in the medium to the long run fiscal prudence is safeguarded under the Godley-Minsky fiscal rule.

Interestingly enough, the simulations in Fig. 5 indicate that the Godley-Minsky fiscal rule is not stabilising when $\theta_1 = 0$, i.e. when the target net debt ratio of the private sector does not change endogenously. The reason is that Fig. 5 refers to a private sector that has a net creditor benchmark position. As mentioned above, in this case a higher (lower) growth rate places upward (downward) pressures on the net debt ratio. Hence, the counter-cyclical effects of the Godley-Minsky fiscal rule are not conducive to stability. This, however, does not hold when the private sector has a net debtor benchmark position ($d_p^B > 0$); in this case the Goldey-Minsky fiscal rule is stabilising even when $\theta_1 = 0$. 
Fig. 5. Dynamic adjustments of the 5D system to a 5% increase in the net private debt-to-income ratio for varying values of $\theta_1$, private sector in a net creditor benchmark position ($d_p^B < 0$), Godley-Minsky fiscal rule.

(a) Net private debt-to-income ratio ($d_p$) 

(b) Private sector’s propensity to spend ($p$)

(c) Target net private debt-to-income ratio ($d_p^T$)

(d) Net government debt-to-output ratio ($d_G$)

(e) Growth rate ($g_Y$)

(f) Government balance-to-output ratio ($b_G$)
Fig. 6 compares the relationship of the private with the government net debt ratio between an economy that implements the Maastricht-type rule and an economy in which government expenditures change according to the Godley-Minsky rule. It can be observed that the nature of the debt cycles is very different in these economies. In the economy that implements the Maastricht-type rule, there are periods in which both the private and the government net debt ratio decline. These ostensibly stable periods of declining net indebtedness are, though, followed by periods where both the government and the private net debt ratio increase. Contrariwise, under the Godley-Minsky fiscal rule the relationship between the two ratios is always inverse. In periods in which the net private debt ratio declines (increases), the net government debt ratio increases (declines). This is the consequence of the attempts of the fiscal authorities to mitigate the contractionary (expansionary) effects that stem for the desire of the private sector to reduce (increase) its indebtedness.

![Fig. 6. Relationship between net private and net government indebtedness in the 5D system, private sector in a net creditor benchmark position \( d^p_d < 0 \), \( \theta_1 = 0.6 \).](image)

(a) Maastricht-type fiscal rule  
(b) Godley-Minsky fiscal rule

6. Concluding remarks

This paper developed a simple macrodynamic model that synthesises certain aspects of Godley’s and Minsky’s analytical frameworks. Within the skeleton developed in the paper it was shown that unstable debt and economic cycles can emerge as a result of the endogenous responsiveness of private sector’s propensity to spend to divergences between actual and target net private debt ratios. In particular, this is the case when the private sector has a net debtor benchmark position. A ‘paradox of
debt’ result was that the more the private sector and its lenders attempt to put net private indebtedness under control by adjusting private expenditures the more the net private debt ratio destabilises. This result is associated with the economic growth consequences of this adjustment which have destabilising feedback effects on net private indebtedness.

A principal outcome of the analysis was that the endogenous changes in the target net private debt ratio during economic cycles always reinforce the destabilising forces in the macro system. Hence, the alternations in the stock-flow norms via the Minsky mechanism put forward in the paper can be an important source of instability.

The implications of the developed framework for fiscal policy were examined. The paper compared the (de)stabilising effects of two different fiscal rules. The first rule is a Maastricht-type fiscal rule according to which the government expenditures-to-output ratio decreases (increases) when the net government debt ratio is higher (lower) than a target level. The second rule is a Godley-Minsky fiscal rule which states that fiscal authorities should increase (decrease) government expenditures when the private sector attempts to decrease (increase) its indebtedness. Simulation analysis illustrated that the Maastricht-type fiscal rule is destabilising while the Godley-Minsky fiscal rule is stabilising. The ‘paradox of debt’ appears to apply to the government sector: the more the fiscal authorities attempt to target a specific government debt ratio by changing the government expenditures the more this ratio destabilises. Moreover, the two fiscal rules produced quite different results as far the relationship between the private and the government net debt ratio is concerned. Under the Maastricht-type fiscal rule, there are periods in which these debt ratios move together producing both euphoric times of decreasing indebtedness and turbulent times of declining indebtedness. Under the Godley-Minsky fiscal rule, the net government debt ratio moves inversely with the net private debt ratio and, therefore, there are no periods in which private and government net indebtedness both increase.
The model of this paper and the results presented above bring to the fore the importance of Godley’s and Minsky’s views about the inherent instability of the macroeconomy and the stabilising role of fiscal policy. Based on these views, the paper provided a new look at the dynamics of the modern macroeconomies. An important line of research would be to combine the Godley-Minsky cycles produced here, which focus on role of debt, with the traditional Goodwin cycles, that concentrate on the role of income distribution.
References


32


Appendix A. Parameter values in the simulations.

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<table>
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<tr>
<td>( \tau )</td>
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</tr>
<tr>
<td>( r )</td>
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</tr>
<tr>
<td>( g_X )</td>
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</tr>
<tr>
<td>( m )</td>
<td>0.3</td>
</tr>
<tr>
<td>( \theta_2 )</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ \lambda = 0.05 \quad d_G^T = 0.5 \]
\[ \tau = 0.4 \quad \mu = 0.034 \]
\[ r = 0.02 \quad \kappa = 0.05 \]
\[ g_X = 0.024 \quad d_B^B = 0.25 \text{ (net debtor)} \]
\[ m = 0.3 \quad d_B^B = -0.25 \text{ (net creditor)} \]
\[ \theta_2 = 0.1 \]
Appendix B. Unstable cycles in the 2D subsystem.

In the 2D subsystem unstable cycles emerge when the fixed point is an unstable spiral node. This holds when \( \lambda > \lambda^* \) and \( \Delta(J_{2D}) < 0 \).

We have that:

\[
\Delta(J_{2D}) = Tr(J_{2D})^2 - 4\text{Det}(J_{2D}) = \left[ r - g_x + \frac{d_R^p(1-\tau)\lambda}{1 + m - p_0(1-\tau) - g_0} \right]^2 - 4\lambda
\]

Equivalently:

\[
\Delta(J_{2D}) = A\lambda^2 + B\lambda + \Gamma
\]

where:

\[
A = \frac{(d_R^p)^2(1-\tau)^2}{(1 + m - p_0(1-\tau) - g_0)^2} > 0
\]

\[
B = \frac{2(r - g_x)d_R^p(1-\tau)}{1 + m - p_0(1-\tau) - g_0} - 4 < 0
\]

\[
\Gamma = (r - g_x)^2 > 0
\]

The roots of \( \Delta(J_{2D}) \) are the following:

\[
\lambda_{1,2} = \frac{-B \pm \sqrt{B^2 - 4A\Gamma}}{2A}
\]

It can be shown that \( B^2 - 4A\Gamma > 0 \) which implies that \( \lambda_{1,2} \) are real.

It holds that:

\[
\Delta(J_{2D}) = A(\lambda - \lambda_1)(\lambda - \lambda_2)
\]
According to Vieta’s formulas, \( \lambda_1 + \lambda_2 = -\frac{B}{A} > 0 \) and \( \lambda_1 \lambda_2 = \frac{\Gamma}{A} > 0 \). Therefore, the two roots are both positive. Assuming that \( \lambda_1 < \lambda_2 \), we have that \( \Delta(J_{2D}) < 0 \) if \( \lambda_1 < \lambda < \lambda_2 \). It can be shown that \( \lambda_1 < \lambda^* < \lambda_2 \). Therefore, unstable cycles occur for \( \lambda^* < \lambda < \lambda_2 \).