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Classical and Keynesian Models of Inequality and Stagnation

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Classical and Keynesian Models of Inequality and Stagnation

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Abstract

This paper studies two formal models of long run growth with a medium-run distributive cycle, both of which feature causal links from the rise in inequality to a deterioration of long run macroeconomic performance. Both versions feature an endogenous income-capital ratio: one through the Keynesian notion of effective demand, the other building on induced bias in technical change. A key focus of the analysis is on the assumptions necessary in both frameworks to generate policy implications consistent with the observed decline of the labor share, the income-capital ratio, and labor productivity growth during the neoliberal era. Importantly, both theories: (a) provide space for mutually reinforcing pro-labor and pro-growth policies in the long run, although they differ in the mechanisms at play in these processes; (b) imply a potential tradeoff between pro-labor policies and growth on one hand, and long-run employment on the other; (c) are consistent with the evidence on the distributive cycle at business cycle frequency.

Keywords: Distributive cycle, induced technical change, labor share, stagnation
JEL Codes: E11, E12, E25, E32, O33, O41

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1 Introduction

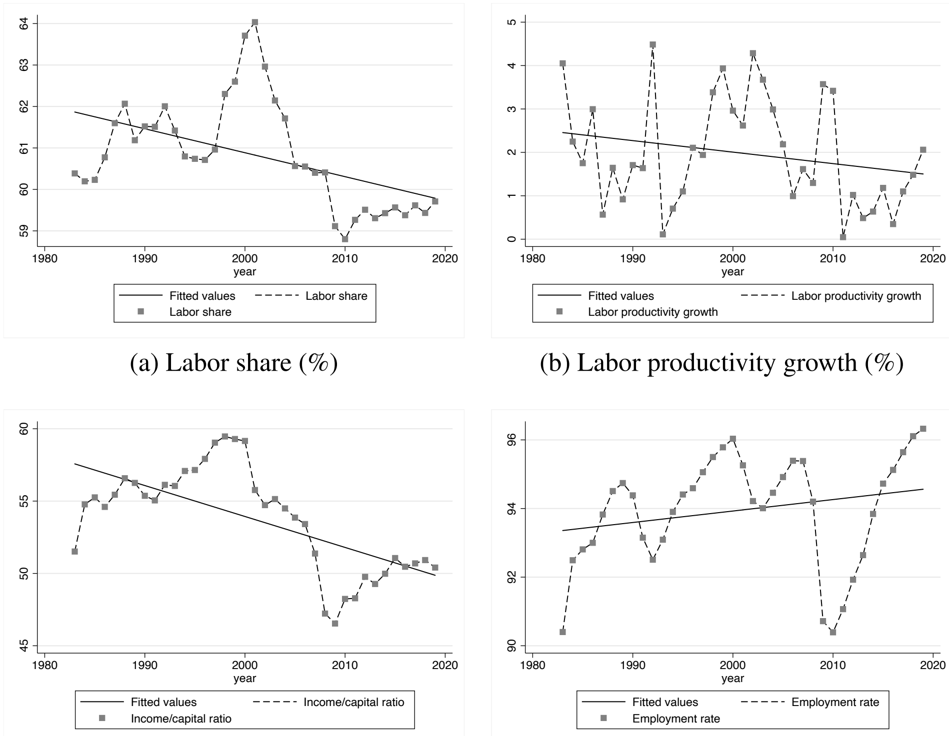
The Great Recession and the slow recovery that followed revived debates about secular stagnation and its relation with distributive variables. The decline in long run macroeconomic performance, however, precedes 2008: as shown in Figure 1, the economic trajectory of the United States after the Volcker disinflationary shock of the early 1980s is characterized by a decline in the labor share of income, a decline in labor productivity growth, and a decline in the income-capital ratio, the latter defined as the ratio of real value added over net fixed assets. Over the same period, the complement to one of the civilian unemployment rate shows a slightly positive trend, despite its volatility due to two deep recessions: the Volcker shock of the early 1980s, and the Great Recession of 2008.[1]

Well-known explanations of the above stylized facts have been advanced by Piketty (2014) and Gordon (2015) using a standard neoclassical growth model with high elasticity of substitution between capital and labor. An exogenous decline in labor productivity growth, which anchors the long-run growth rate of the economy, is responsible for an increase in the difference between the rate of return to wealth r and the growth rate g. Accordingly, the capital-income ratio rises (the income-capital ratio falls). If the substitution elasticity exceeds unity, the decline in the income-capital ratio will result in a falling labor share of income. The empirical exercise presented in Karabarbounis and Neiman (2014) provided support for the requirement of an elasticity of substitution above one for a cross-section of countries. Importantly, all of these accounts presuppose full employment at all times: they cannot explain the trend in the (un)employment rate illustrated above.

The linkages between income shares and macroeconomic performance, both in levels (the income-capital ratio) and growth rates has long been a focus of heterodox macroeconomics. This literature rejects instantaneous, smooth factor substitution, marginal productivity pricing and clearing labor markets, and hence does not presuppose full employment. Starting with the contributions by Rowthorn (1981) and Dutt (1984), authors in the neo-Kaleckian tradition have advanced the idea that the income-capital ratio as a proxy for aggregate demand—and possibly the growth rate, depending on model specification—is wage-led through the role of consumption out of wages in boosting effective demand. Importantly, the labor share is seen as fully exogenous in this literature. Labor-friendly redistribution policies would then have positive effects on economic activity (see also Taylor, 1983; Amadeo, 1986).

However, the available empirical evidence on the systematic cyclical interactions between measures of economic activity and income shares in the US is not favorable to the neo-Kaleckian model. First, the observed *distributive cycle*, characterized by counterclockwise fluctuations in macroeconomic activity *and* the labor share, requires both variables to be endogenous (Barrales and von Arnim, 2017). Further, an array of empirical research

[1] Li and Mendieta-Muñoz (2020) provide related references, discussion and empirical evidence that the decline in (G-7) growth rates post-2008 is *not* driven by recessionary factors, but stagnationary tendencies that preceded the crisis.



(a) Labor share (%)

(b) Labor productivity growth (%)

(c) VA/NFA (%)

(d) Employment rate (%)

Figure 1: **US macroeconomic trends, 1983-2019.** Sources: FRED (Labor share, labor productivity growth and employment rate); BEA (VA/NFA).

demonstrates that aggregate demand is *profit-led*, and that activity leads the cycle, contrary to the implications of the neo-Kaleckian model.[2]

That aggregate demand appears to be profit-led has been seen by neo-Kaleckians as the ultimate defeat of progressive redistribution policies. And yet, recent theoretical and applied work in both Classical and Keynesian traditions has advanced the hypothesis that labor-friendly policies may be beneficial to long-run growth through supply forces, i.e. *induced or endogenous technical change* (see Tavani and Zamparelli, 2017, for a comprehensive survey). Policy-driven increases in real wages relative to labor productivity may spur

[2]The empirical literature is voluminous and we will not review it extensively. Barbosa-Filho and Taylor (2006); Proaño et al. (2006); Flaschel et al. (2006); Tavani *et al.* (2011); Rada and Kiefer (2015); Barrales and von Arnim (2017); Basu and Gautham (2019) provide evidence in favor of profit-led activity and profit-squeeze distribution, and Barrales *et al.* (2022) confirm and survey this consensus. Stockhammer and Michell (2017) have suggested the theoretical possibility of "pseudo-"Goodwin cycles with wage-led demand when a financial cycle is introduced. Barrales *et al.* (2022) find evidence that demand is profit-led even when including a financial cycle, thus rebutting their theoretical argument on empirical grounds. Current research concerns the apparent weakening of the mechanisms underlying the Goodwin pattern; see Mendieta-Muñoz et al. (2020) for empirical results on this issue and Setterfield (2021) for related discussion.

labor-saving innovations that will ultimately generate faster labor productivity growth.[3]

Our goal in this paper is to address this tension between (short run) profit-led activity and (long run) wage-led productivity growth. To do so, we present two formal macrodynamic models of long-run growth and distribution around an extended Goodwin (1967) distributive cycle. Our contribution is to delineate necessary assumptions, relevant mechanisms, and key results. In both models, a deterioration of labor market institutions and the consequent increase in inequality leads to a rise in demand and accumulation in the short run. However, long-run macroeconomic performance ultimately worsens, both in levels (i.e. a decline in the income-capital ratio) and in growth rates (i.e. a decline in output and labor productivity growth). Key to both models is that they feature an endogenous income-capital ratio in addition to income shares and employment rate. However, the main mechanisms at play are starkly different.

The model of the classical distributive cycle (CDC, hereafter) features the notion of induced technical change as in the work of Kennedy and Weizsäcker (see footnote 3). As in Shah and Desai (1981), the income-capital ratio adjusts as the result of the firm's profit-maximizing choice of the direction of technical change, and in the process stabilizes the Goodwinian capital-labor conflict. The Keynesian distributive cycle (KDC hereafter) version hinges instead on the notion of effective demand. Quantity adjustments ensure equilibrium in the goods market, and technical change is endogenous to—and increasing in—the labor share. Despite these differences, both models imply a long run decline in the income-capital ratio: a deterioration of labor market institutions increases profitability and puts upward pressure on the accumulation rate, but the long run growth rate of labor productivity has fallen. Restoring balanced growth requires the income-capital ratio to decline.

The final portion of our argument concerns the relation between the functional distribution of income and long run employment. The Keynesian model features an unambiguous tradeoff between labor-friendly redistribution and the long run employment rate. While the Classical model is in principle ambiguous in this regard, trends in the United States economy during the neoliberal era suggest the distribution-employment tradeoff to be relevant (see Figure 1).

Our analysis builds on and extends the many fundamental contributions made by the late Peter Flaschel on these issues over his long and illustrious career. First, and as in Flaschel (1993, 2009), our research emphasizes not only policy effects in steady state, but also disequilibrium processes, transitional dynamics and fluctuations around a balanced growth path. Second, and similarly to Chiarella and Flaschel (2000); Flaschel (2015), we

[3]The literature builds on Kennedy (1964) and von Weizsäcker (1966). Shah and Desai (1981); Foley (2003); Julius (2005); Rada (2012); Zamparelli (2015) incorporate this mechanism of induced technical change in classical Goodwinian frameworks. Keynesian and structuralist research draws on the underlying idea—that high real unit labor costs spur labor-saving technical change—but does not utilize optimizing, supply-driven model structures (Barbosa-Filho and Taylor, 2006; Storm and Naastepad, 2012). Rada et al. (2021, 2022) elaborate the relevant theoretical linkages.

present both supply- and demand-driven models of an extended Goodwin (1967) growth cycle, recognizing the central importance of the framework for heterodox macroeconomics. Third, our focus on the potential trade-off between employment and distribution connects this effort to arguments about labor market institutions and flexicurity advanced in Flaschel and Luchtenberg (2013).

We proceed as follows. The next section introduces common elements of the two models. Section 3 and Section 4 present KDC and CDC models and their comparative statics implications, respectively. The following Section 5 illustrates transitional dynamics of both models with numerical simulations. The last section concludes. To foreshadow results, (I) at business cycle frequency, both models imply that income-capital ratio and employment rate lead the labor share, and thus conform to the empirical evidence (see footnote 2). Further, (II) in steady state of both models, weaker labor market institutions reduce the labor share, the income-capital ratio *and* the growth rate of labor productivity, thus producing key stylized facts of the neoliberal era. Throughout, (III) we discuss in detail the necessary assumptions. Our main contribution thus can be summarized as providing two theories—different in specific mechanism, similar in foundational ideas—that render capital-friendly labor market policies a cause of inequality and stagnation.

2 Common elements

The purpose of this section is to introduce notation, and provide a common framework for the exposition of KDC and CDC models subsequently. Key similarities between the two frameworks pertain to (i) production technology, (ii) class-based savings behavior, (iii) the theoretical core of Goodwin's distributive growth cycle, and (iv) extensions of the latter to endogenize the income-capital ratio.

We begin with the production technology. In both the Keynesian and the Classical model, final output Y is produced using fixed proportions of capital K and labor L. We assume in standard fashion that the labor force N grows exogenously at a rate $n > 0$. Denoting the existing stock of labor-augmenting technologies by A, the output-capital ratio (or, equivalently, income-capital ratio) at full utilization by $\sigma \equiv Y^p/K$, the rate of utilization by $U \equiv Y/Y^p$, and the observed output-capital ratio as $u = \sigma U$, the aggregate production technology is

$$Y = \min\{AL, \sigma U K\}. \tag{2.1}$$

CDC and KDC differ fundamentally in their approach to the income-capital ratio. The CDC model assumes continuous full utilization. For brevity, we set $U = 1$. Accordingly, Section 4 introduces the income-capital ratio directly as σ, which—as will be discussed further below—is determined by technology choices made by firms in the economy. In contrast, the KDC model assumes continuous *under*-utilization of capital. For brevity, we set $\sigma = 1$. Hence, in Section 3 the observed income-capital ratio u is a state variable, and implicitly dominated by demand variation via the rate of utilization U. In short, both frameworks model the realized income-capital ratio, but in CDC the technical coefficient σ becomes endogenous, whereas in KDC the rate of utilization becomes endogenous.

Next, we consider class-based saving behavior and related accounting. In both models, the economy is populated by two classes of households: capitalists own the means of production, receive profit income Π after paying wages to workers, and save a constant fraction of profits denoted by $s_\pi \in (0,1)$. Workers supply labor to firms, earn a real wage Ω, and do not save. Given that profit-maximization requires firms to set effective capital σUK equal to effective labor AL, with wage share $\psi \equiv \Omega L/Y = \Omega/A$, capitalists' profits will be $\Pi = \sigma U(1-\psi)K = u(1-\psi)K$—with $U = 1$ in the Classical model and $\sigma = 1$ in the Keynesian model—and total savings in the economy will be $s_\pi \Pi$. Denoting the profit share as $\pi = 1 - \psi$, the profit rate follows as $r = \pi\sigma U = \pi u$.

Both models build on the theoretical core of the distributive cycle, which pits the employment rate $e \equiv L/N = Y/(AN)$ and the labor share as respectively the prey and the predator in a conflictual yet symbiotic relationship. As in Goodwin (1967), log-differentiation of state variable definitions implies the standard model structure as

$$\hat{e} = \hat{Y} - (\hat{A} + n) \tag{2.2}$$
$$\hat{\psi} = \hat{\Omega} - \hat{A}. \tag{2.3}$$

At steady state, $\hat{e} = \hat{\psi} = 0$. In consequence, the growth rate of output must be equal to the natural rate of growth. In this sense, both Goodwinian models presented here are labor-constrained: the conflict over the functional distribution of income maintains a constant employment rate in the long run. It follows that the steady state growth rate of output converges to the growth rate of the effective labor force, or Harrod's natural rate. Importantly, a constant employment rate in steady state does *not* imply full employment: even in the classical version, where Goodwin's core assumes Say's Law, the labor market does not necessarily clear. For future reference, we define the warranted and natural rate:

$$g^w = s_\pi(1-\psi)\sigma U \tag{2.4}$$
$$g^* = \hat{A} + n. \tag{2.5}$$

Further, both models incorporate the income-capital ratio as a third state variable. As already discussed, in the CDC model, the full capacity income-capital ratio σ becomes endogenous to the evolution of technology implied by the firms' profit-maximizing behavior, whereas in the KDC model the observed income-capital ratio u is determined through a multiplier-accelerator process.

Finally, in both models we investigate how changes in labor market institutions affect the macroeconomy in the short and the long run. To that end, we introduce the parameter z to measure the quality of labor market institutions. As will be discussed in more detail below, the parameter z enters the real wage Phillips curve in the Keynesian and classical model, but in the latter additionally affects the innovation possibility frontier, which constrains the choice of technological improvements made by firms and is carefully described below. By convention, we assume that an increase in z captures a labor-friendly shift in labor market institutions.

3 The Keynesian model

This section presents the cyclical and long-run dynamics of a Keynesian model of the distributive cycle. Effective demand, its main feature, is introduced via an independent output growth function $h = \hat{Y}$ embedded in the laws of motion of income-capital ratio and employment rate. Differently from the Classical model below, changes in the income-capital ratio do not originate in technology and firm's optimization behavior, but in the mechanism driving the equalization of the growth of expenditures (i.e., demand) and capital stock (i.e., supply). This closure of the model allows us, as outlined in the previous section, to simply write the observed income-capital ratio as $u = Y/K$. Note that the steady-state income-capital ratio in both models is constant and in line with Harrod-neutral technical change. Crucially, and differently from the Classical model, the disequilibrium between output growth and capacity growth emerges here due to the independent role of aggregate demand, and it is addressed via quantity-adjustment in u.

At business cycle frequency, the model exhibits profit-led activity and profit-squeeze distribution: that is, demand varies inversely with the labor share, and the labor share increases in the rate of capacity utilization. The former feature emerges from the following mechanisms, discussed in more detail below: first, a rise in the profit share stimulates investment demand and therefore output growth while all along increasing capacity and consequently the warranted growth rate via saving. Second, we consider the employment rate as another measure of economic activity, which, as is common in Keynesian approaches, varies directly with aggregate demand and, therefore, with the income-capital ratio. Finally, we postulate in standard fashion that rising employment increases the real wage (and, hence, *ceteris paribus*, increases the labor share) and squeezes profits. In a nutshell, over business cycles, economic activity leads distribution.

Distributive conflict takes center stage in the long run via the induced technical change effect. This effect allows a labor market shock to the real wage, and therefore to the labor share, to reverberate into the growth rates of labor productivity and output. The endogenous labor productivity growth ensures that labor supply does not constrain growth in the long run, and reconciles the demand-determined rate of output growth with the natural rate of growth.

3.1 Behavioral functions

The model is built on Classical and Keynesian foundations that feature the warranted rate of growth g^w (equation 2.4) and three behavioral functions. These describe the growth of output via an independent expenditure function h, endogenous labor productivity growth a, and the real wage Phillips curve ω:

$$\hat{Y} = h(u, e, \psi), h_u > 0, h_e < 0, h_\psi < 0 \tag{3.1}$$

$$\hat{A} = a(\psi), a_\psi > 0 \tag{3.2}$$

$$\hat{\Omega} = \omega(e, z), \omega_e > 0, \omega_z > 0. \tag{3.3}$$

The partials in equations 3.1-3.3 can be motivated as follows. First, and broadly in line with Skott (1989)'s pioneering work on cyclical growth, h_u is positive: a higher level of demand as proxied by a higher income-capital ratio u leads to an increase in the growth rate of output. This positive sign ($h_u > 0$) also assures that increases in demand u lead to increases in the employment rate e, which in turn ensures the stability of the overall dynamics of the economy.

Second, a tight labor market captured by a higher rate of employment e makes the expansion of production more costly. Skott (1989, p. 236) motivates the sign for the partial, $h_e < 0$ as a decrease in the desired rate of expansion due to adjustment and turnover costs at high employment rates. The sign can also be motivated with direct reference to Kalecki (1943)'s seminal essay on the "political aspects of full employment:" high employment rates undermine the power of capital, and thus depress expansion plans (see also Flaschel and Skott, 2006; Flaschel et al., 2008).

Further, $h_\psi < 0$ represents a Kaleckian link from the functional distribution of income to economic activity. The built-in assumption is that investment demand responds negatively to a higher labor share, *and* does so sufficiently strongly to overcome any positive effects of ψ on consumption expenditures.

The induced technical change effect ($a_\psi > 0$) implies that higher real wages relative to labor productivity trigger efforts to economize on labor costs. As will be seen in Section 4, this positive relation can be formally micro-founded through the profit-maximizing choice of the direction of technical change by competitive firms (see also footnote 3). Here we model aggregate productivity directly, and simply assume that pressure for labor-saving innovation arises when real unit labor costs are high: $\hat{A} = a(\psi), a' > 0$. For further discussion of this reduced-form approach, see Barbosa-Filho and Taylor (2006); Storm and Naastepad (2012).

Finally, real wage growth responds to the employment rate and the quality of labor market institutions according to a real wage Phillips curve: $\hat{\Omega} = \omega(e, z)$, with $\omega_e, \omega_z > 0$. The profit squeeze arises as the ultimate result of a Keynesian chain of causation: high demand increases the income-capital ratio u, which drives up the employment rate e, which in turn puts upward pressure on real wage growth and hence the labor share ψ. This mechanism is common in neo-Goodwinian models and related empirical applications.[4]

3.2 The dynamical system

The model consists of three differential equations describing the evolution of the income-capital ratio, the employment rate, and the wage share. Given the definition of u, e and ψ, log-differentiation provides the following laws of motion for the three state variables:

$$\dot{u} = u(\hat{Y} - \hat{K}) = u[h(u, \psi, e) - s_\pi(1-\psi)u] \tag{3.4}$$
$$\dot{e} = e[\hat{Y} - (\hat{A} + n)] = e[h(u, \psi, e) - a(\psi) - n] \tag{3.5}$$
$$\dot{\psi} = \psi(\hat{\Omega} - \hat{A}) = \psi[\omega(e, z) - a(\psi)]. \tag{3.6}$$

In the non-trivial steady state, $\dot{u} = \dot{e} = \dot{\psi} = 0$, and the three state variables attain their non-zero steady state levels, indicated by a star. This also implies that $\hat{Y} = h = g^w = g^* = \hat{A} + n$, and $\hat{\Omega} = \hat{A}$. Put differently, in steady state, Harrod's three growth rates equilibrate, and Kaldor's stylized facts are satisfied. The steady state of the system above with generic functions can be characterized in reduced form as follows:

$$u^* = \Phi^u(\boldsymbol{\alpha}, z) \tag{3.7}$$
$$e^* = \Phi^e(\boldsymbol{\alpha}, z) \tag{3.8}$$
$$\psi^* = \Phi^\psi(\boldsymbol{\alpha}, z) \tag{3.9}$$

where, for example, Φ^u denotes that the steady state income-capital ratio u^* is a function of a set of parameters $\boldsymbol{\alpha}$ and the critical parameter z. Crucially, z appears only in the real wage Phillips curve, and directly affects the steady state labor share via equation 3.6. Further, the income-capital ratio adjusts to equalize the growth rate of output h with the warranted rate g^w, the employment rate adjusts to equalize the growth rate of output h with the natural rate g^*, and the labor share adjusts to equalize the growth rates of real wage and labor productivity.

The Jacobian matrix of this system, evaluated at the steady state (though without starring of variables, for the sake of brevity), is

$$J^* = \begin{bmatrix} u(h_u - s_\pi(1-\psi)) & uh_e & u(h_\psi + s_\pi u) \\ eh_u & eh_e & e(h_\psi - a_\psi) \\ 0 & \psi\omega_e & -\psi a_\psi \end{bmatrix}. \tag{3.10}$$

We assume $h_u < s_\pi(1-\psi)$ and $|h_\psi| > s_\pi u$, which implies that the income-capital ratio is

[4]For a discussion of profit-squeeze effects between labor share on the one hand and income-capital ratio or employment rate on the other, see Diallo et al. (2011). These authors motivate pressure from u on ψ vs. e on ψ on the basis of insider vs. outsider bargaining, respectively. In our model, only outsider bargaining matters, but the resulting dynamics are qualitatively equivalent.

self-stabilizing; and that it reacts negatively to a rising labor share, which implies a profit-led economy. As a result, the determinant is unambiguously negative ($|J^*| < 0$), which is necessary for dynamic stability. See Appendix A.2 for a signed Jacobian matrix, and a proof of local stability. Further, the sign pattern generates relevant cyclical stylized facts (Zipperer and Skott, 2011; von Arnim and Barrales, 2015; Barrales *et al.*, 2022), and the two-dimensional subsystems are consistent with empirically-observed cycles in u, e and e, ψ.[5]

It is worth stressing once again that this three-dimensional model resolves both of Harrod's problems without sacrificing the essential Keynesian property of a long run role for aggregate demand. Importantly, the solution is facilitated by the interaction between the labor constraint and the functional distribution of income in determining the natural rate of growth. Indeed, the crux of the matter is that the growth rate of output equilibrates with the warranted rate of growth (in equation 3.4) and, additionally, with the natural rate of growth (in equation 3.5). This of course implies also that $g^w = g^*$. At the same time, the labor share is tied to institutions governing real wage bargaining as described by z, so that:

$$\frac{\partial g^*}{\partial z} = a_\psi \frac{\partial \psi}{\partial z} > 0 \Leftrightarrow \frac{\partial g^w}{\partial z} > 0. \tag{3.11}$$

Consider now the effect of an erosion in labor market institutions and bargaining power of workers, captured by a decline in the z-term in the real wage Phillips curve. Recall that $|J^*| < 0$; further, we denote the (i, j)-th minor as $|J_{ij}|$. Cramer's rule then implies

$$\frac{\partial u^*}{\partial z} = -\psi \frac{|J_{31}|}{|J^*|} = h_e(a_\psi + s_\pi u) \frac{eu\psi}{|J^*|} > 0 \tag{3.12}$$

$$\frac{\partial e^*}{\partial z} = \psi \frac{|J_{32}|}{|J^*|} < 0 \tag{3.13}$$

$$\frac{\partial \psi^*}{\partial z} = -\psi \frac{|J_{33}|}{|J^*|} > 0, \tag{3.14}$$

where $|J_{32}|$ and $|J_{33}|$ are straightforward to sign, and $|J_{31}|$ is also unambiguous due to h_e and h_ψ appearing in both columns of the minor. (See Appendix A.2 for details.)

Thus, an adverse shock to labor's bargaining power implies an increase in the employment rate, a fall in the steady-state labor share, a fall in the income-capital ratio, and a decline in the long-run growth rate since $\partial g^*/\partial z = a_\psi \partial \psi/\partial z > 0$. In other words, this shock leads to a new steady-state that features higher inequality, lower growth and higher employment. Thus, the model's key variables match the stylized facts of the neoliberal era.

In summary, in this Keynesian model of the distributive cycle, the labor share is linked in steady state to institutions of the labor market, rather than merely technology. In this

[5] The u, ψ-cycle emerges only in the three-dimensional system, and is there determined by $\partial \dot{e}/\partial u = eh_u > 0$: the employment rate increases in the income-capital ratio, and then drives the profit squeeze via ω_e. See Rada et al. (2021, 2022) for further discussion.

view, the state, asked to retreat in the face of excessive faith in markets, and the neoliberal labor market, deregulated and deskilled to favor capital, join forces to depress labor share, income-capital ratio and steady state growth, but generate countervailing forces on the employment rate.

3.2.1 Short-run effect of a negative shock to z

It is worthwhile to trace out not only the long-run, but also the short-run effects of a negative shock to z on the economy starting from a steady state equilibrium. The institutional variable affects the dynamics of the system merely through its effect on real wage growth. Given the initial level of labor productivity growth, lower wage growth reduces the wage share: $\dot{\psi} < 0$. The fall in the wage share affects the dynamics of both the employment rate and capacity utilization. It simultaneously raises output growth and decreases labor productivity growth; and both effects contribute to higher labor demand, or $\dot{e} > 0$. The effect on capacity utilization, on the other hand, is ambiguous in principle. The lower wage share increases both output growth and the warranted growth rate. The numerical simulations discussed in Section 5 show that the first effect appears to dominate on impact so that the output-capital ratio increases.

4 The Classical model

The Classical version of the model builds on the contributions by Shah and Desai (1981) and Julius (2005), who introduced the induced innovation hypothesis in the Goodwin growth cycle. The main consequence of this integration is that the perpetual cyclical fluctuations in employment rate and labor share are resolved by the induced feedback from the latter to labor productivity growth: while there are fluctuations along the transitional dynamics, the Goodwin steady state becomes ultimately stable. We generalize their contribution with the introduction of labor market institutions, represented by the shift variable z.

4.1 Accumulation, innovation and choice of productivity growth

As discussed in Section 2, the Classical model assumes continuous full capacity utilization so that $U = 1$ and $u = \sigma$. Accordingly, since all savings are automatically invested, capital accumulation is:

$$\hat{K} = s_\pi(1 - \psi)\sigma. \tag{4.1}$$

We model innovation by following the induced technical change literature. The innovation possibility frontier (IPF) describes the evolution of technology by defining the set of growth rates of labor and capital productivity freely available to competitive firms. If we let $a = \hat{A}$,

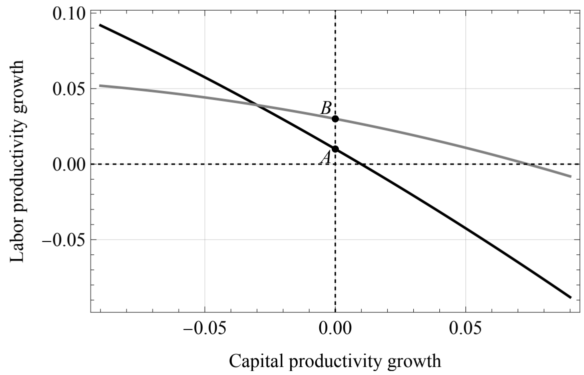


Figure 2: **Innovation Possibility Frontier**. Starting at a baseline Harrod-neutral profile of technical progress (point A), an increase in z under Assumptions 1 and 2 rotates the frontier counterclockwise and makes it flatter at the new, higher Harrod-neutral rate (point B). See Section 5 for the functional form used in order to obtain this plot.

we have $a = f(\hat{\sigma}; z)$, with $f_{\hat{\sigma}} < 0$, $f_{\hat{\sigma}\hat{\sigma}} < 0$. The frontier is decreasing and strictly concave in order to capture the increasing complexity in the trade-off between labor-augmenting and capital-augmenting innovations. With respect to the relation between the IPF and labor market institutions, two assumptions ensure that labor-friendly policies increase long run growth, the wage share, and the output-capital ratio:

Assumption 1 $\frac{\partial f_{\hat{\sigma}}(0,z)}{\partial z} > 0$*: the slope of the IPF is strictly increasing in* z*.*

Assumption 2 $\frac{\partial f(0;z)}{\partial z} > 0$*:* z *positively affects the Harrod-neutral rate of technological progress.*

Assumption 1 requires that a higher z renders the IPF flatter at the vertical intercept. It could be argued that in a more conflictual economic environment trading-off capital-saving with labor-saving technical change may be harder; therefore, the marginal rate of transformation between the two types of innovations becomes smaller. According to Assumption 2, firms manage to implement higher labor productivity growth when workers' bargaining power rises. This could be rationalized as in this context firms may have stronger incentives to replace labor. Figure 2 displays the effect of an increase in the labor market parameter following an increase in z.

We assume that firms choose the direction of technical change, that is a point ($\hat{\sigma}$, a) on the IPF, in order to maximize the instantaneous rate of unit cost reduction $\psi a + (1-\psi)\hat{\sigma} = \psi f(\hat{\sigma}; z) + (1-\psi)\hat{\sigma}$. The first order condition is

$$-f_{\hat{\sigma}}(\hat{\sigma}; z) = \frac{1-\psi}{\psi}. \tag{4.2}$$

Since $f_{\hat{\sigma}\hat{\sigma}} < 0$, capital productivity growth is an inverse function of the wage share, say

$\hat{\sigma} = b(\psi, z), b_\psi < 0$. Therefore, labor productivity growth is a direct function of the wage share: $a = f\left[b(\psi, z); z\right], a_\psi > 0$.

4.2 The dynamical system

The three state variables of the economy are capital productivity (or income-capital ratio), the employment rate, and the wage share. The evolution of capital productivity follows from the firm's first order conditions. In order to find the dynamics of the employment rate, we plug $\hat{Y} = \hat{\sigma} + \hat{K} = b(\psi, z) + s_\pi(1-\psi)\sigma$ and $\hat{A} = f\left[b(\psi, z)\right]$ into equation (2.2). Finally, we need to define real wage growth to track movements in the labor share. In line with the Keynesian model, we assume $\hat{\Omega} = \omega(e, z)$, with $\omega_e, \omega_z > 0$. Accordingly, the dynamical system is:

$$\begin{aligned}\dot{\sigma} &= \sigma b(\psi, z) &(4.3)\\ \dot{e} &= e\left\{b(\psi, z) + s_\pi(1-\psi)\sigma - (f\left[b(\psi, z); z\right] + n)\right\} &(4.4)\\ \dot{\psi} &= \psi\left\{\omega(e, z) - f\left[b(\psi, z); z\right]\right\} &(4.5)\end{aligned}$$

The first equation shows that the IPF is solely responsible for the determination of the wage share in the long run. Remembering that x^* denotes the steady state value of variable x, using equation (4.2), and setting $\dot{\sigma} = \dot{e} = \dot{\psi} = 0$, we obtain the non-trivial steady state as described by the three equations:

$$\begin{aligned}\psi^* &= \frac{1}{1 - f_{\hat{\sigma}}(0; z)}, &(4.6)\\ \sigma^* &= \frac{f(0; z) + n}{s_\pi(1 - 1/(1 - f_{\hat{\sigma}}(0; z))}, &(4.7)\\ \omega(e^*, z) &= f(0; z). &(4.8)\end{aligned}$$

The Jacobian matrix evaluated in steady state (again without starring variables for notational simplicity) is:

$$J = \begin{bmatrix} 0 & e(b_\psi - s\sigma - a_\psi) & es_\pi(1-\psi) \\ \psi\omega_e & -a_\psi & 0 \\ 0 & \sigma b_\psi & 0 \end{bmatrix}, \tag{4.9}$$

and Appendix A.3 shows the signed Jacobian matrix and provides a proof of the local stability of the steady state.

We now focus on the comparative dynamics of the model with respect to z. Equation (4.6) illustrates that the long run functional distribution of income is fully determined by the slope of the IPF at the steady state. Given our Assumption 1, a positive shock to z raises the wage share. The IPF and z are also the only determinants of long run growth as $a^* = f(0; z)$. The steady state labor productivity growth is the vertical intercept of the innovation possibility frontier and, given Assumption 2, it moves up with z: see the increase from A to B in Figure 2. The steady state income-capital ratio ensures the equality between the natural $(a + n)$ and the warranted growth rate $\hat{K}$. Under the assumptions discussed, worker-friendly labor market policies affect the income-capital ratio in two ways. They raise labor productivity growth and, in turn, the natural growth rate, while they harm total saving and the warranted growth rate through the negative impact on the profit share. Equation (4.7) shows that both effects contribute to a rise in the income-capital ratio. The steady state employment rate, on the other hand, stabilizes the wage share dynamics by ensuring that real wage growth equals labor productivity growth. A change in labor market institutions will have an ambiguous effect on the employment rate if z simultaneously raises wage and labor productivity growth. Equation (4.8) shows that employment will fall when $\omega_z(e, z) > f_z(0; z)$, so that a lower employment rate is necessary to slow down wage growth. This seems the relevant case compatible with the empirical evidence discussed in the Introduction.

4.2.1 Short-run Effect of a Negative Shock to z

Let us now follow the effects of a negative shock to z on the economy starting from a steady state equilibrium. As already discussed, the institutional variable affects the dynamics of the system through three separate effects: (a) wage growth; (b) shape, and (c) position of the IPF. The first step consists in understanding what happens to capital productivity growth $\hat{\sigma}$. The firm's optimization implies that equation (4.2) is continuously satisfied. We can totally differentiate it to find $d\hat{\sigma}/dz = b_z = -f_{\hat{\sigma},z}/f_{\hat{\sigma},\hat{\sigma}} > 0$, under Assumption 1 and given the concavity of the IPF. So we know that capital productivity growth decreases on impact after a negative shock to labor market institutions. The effect on labor productivity growth is less obvious at first sight. A movement along the IPF induced by $d\hat{\sigma} < 0$ tends to increase the growth rate a, but the inward shift of the frontier acts in the opposite direction: in fact $da/dz = \frac{\partial f}{\partial \hat{\sigma}} \frac{d\hat{\sigma}}{dz} + \frac{\partial f}{\partial z}$ where the two terms of the sum have opposite signs. We prove in Appendix A.3.1, however, that in order to satisfy (4.2), firms will respond to a decrease in z by initially increasing labor productivity growth while simultaneously decreasing capital productivity growth. This means that on impact technical change is Marx-biased, with negative capital productivity growth while positive labor productivity growth (Foley et al., 2019). Furthermore, wage growth decreases because of the effect of the institutional parameter on the wage-Phillips curve. These conclusions imply that all three state variables of the dynamical system formed by equations (4.3), (4.5) and (4.4) decline on impact: the wage share declines given the joint decrease in real wage growth and increase in labor productivity growth; the income-capital ratio falls as capital productivity growth becomes negative; and, finally, the employment rate decreases due to the simultaneous fall in capital productivity and warranted growth rate and rise in labor productivity growth. This results —which, once again, always holds under the assumptions made in this model— are illustrated in the numerical simulations discussed in the next Section.

5 Transitional dynamics and numerical simulations

This section presents numerical simulations to illustrate the transitional dynamics of both models. First, simulations confirm that both models portray cyclical dynamics of the Goodwin-type: both activity variables (employment rate e and income-capital ratio σU) lead the labor share ψ.

Second, the focus lies on the response of both models to a decline in the parameter z. As previously discussed, this parameter captures characteristics of labor market institutions, and we assume that a decline in z renders these institutions more friendly to capital. Further, simulations demonstrate (i) the effect of a change in z *on impact*, and (ii) the very different effects of a change in z between short run and long run (i.e., steady state). In particular, (i) differs across the two models, since the firms' optimization problem in the classical model requires instantaneous adjustments so that (4.2) remains continuously satisfied. Most importantly for our purposes, (ii) shows that capital-friendly labor market policies lead to a boom in accumulation in the short run, but imply stagnation in the long run.

It should be emphasized that these simulations are merely illustrative. We calibrate the steady states of income-capital ratio, employment rate and labor share to plausible values for an advanced capitalist country such as the US, and assume signs of key coefficients consistent with available empirical evidence (again for the US). However, we are not conducting exercises to match higher moments, or estimate the model.

5.1 Calibration

We begin with an overview of the common elements for both models: the three state variables' steady states, the warranted growth rate and the real wage Phillips curve. Steady state values of income-capital ratio, employment rate and labor share are $u = \sigma U = 0.4$, $e = 0.9$ and $\psi = 2/3$. These roughly correspond to longer run averages for the US macroeconomy. The warranted rate of growth is $g^w = s_\pi(1-\psi)\sigma U$, and equal to realized rate of growth g and natural rate g^* in steady state. We assume that the steady state rate of growth is $g = g^w = g^* = 0.03$, which, given ψ and u implies $s_\pi = 0.225$. Further, the growth rate of the labor force is constant ($n = 0.01$), so that in steady state $\hat{A} = a = 0.02$.

The real wage Phillips curve also appears in both models. It is implemented as a linear function of the employment rate,

$$\omega(e, z) = \omega_0 + ze, \tag{5.1}$$

where $z = 1$ is the critical parameter describing labor market institutions. The calibration exercise leads to $\omega_0 = -0.88$ given e and z, and that at the steady-state $\omega = a = 0.02$.

The key differences are rooted in the different behavioral functions of the CDC and KDC model. The CDC model is built around the innovation possibility frontier (IPF), whereas the KDC model draws on reduced-form functions for output growth h and labor productivity growth a. We discuss these in turn. A functional form for the IPF that satisfies all the necessary assumptions stated previously is

$$\hat{A} = f[b(\psi, z)] = -b^{\beta} - \alpha \frac{b}{z} + z a^{*}, \tag{5.2}$$

where we set $\beta = 2, \alpha = 1/2$. With $z = 1$ and $a^{*} = 0.02$, this parabola conforms to Harrod-neutral growth in steady state as outlined just above, when $\dot{\sigma} = 0$. Further, a decline in z rotates the IPF clockwise around a point in the North-Western quadrant with negative capital productivity growth and positive labor productivity growth.[6] The first-order condition (4.2) and this functional form for the IPF imply

$$\hat{\sigma} = b(\psi, z) = \left(\frac{1}{\beta} \left(\frac{1}{\psi} - \frac{\alpha}{z} - 1 \right) \right)^{\frac{1}{\beta - 1}}, \tag{5.3}$$

which fully determines the steady state labor share as $\psi = \frac{z}{z+\alpha} = 2/3$. Note that equation 5.3 determines capital productivity growth, and, after substitution in 5.2, also labor productivity growth. Output growth follows as the sum of capital productivity growth b and the warranted growth rate g^{w}.

In the KDC model, output growth is determined by the function h in equation 3.1, and labor productivity growth by the function a in equation 3.2. To match the signs assumed for the Jacobian at the steady state, see equation 3.10 and Appendix A, we assume linear functions and calibrate these as follows:

$$h = h_0 + h_u u + h_e e + h_{\psi} = 0.22 + 0.065u - 0.02e - 0.3\psi \tag{5.4}$$

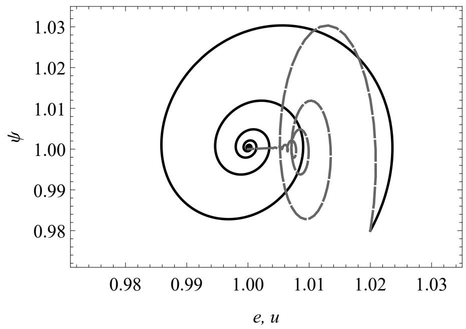
$$a = a_0 + a_{\psi} \psi = -0.15 + 0.25\psi. \tag{5.5}$$

To summarize, we note that with these parameters and in steady state, the Jacobian matrices of both models show the sign patterns as noted in Appendix A, and both models feature a pair of complex eigenvalues with negative real parts, and a third real and negative eigenvalue. Accordingly, both models display damped oscillations around their steady state values.

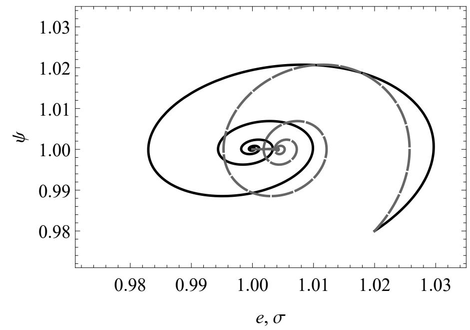
5.2 Discussion

This section discusses the results of the simulation exercises on the basis of three different figures. Figure 3 shows trajectories in the phase space. Figures 4 and 5 report time paths of state variables and key growth rates, respectively, in response to an assumed 2% decline in the parameter z. Across all of these figures, KDC model output is shown in the left column

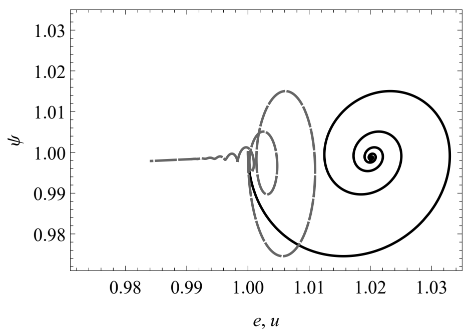
[6] A visual illustration is provided in Figure 2: starting from the IPF in black that intersects the vertical axis at point B, a decline in z produces a clockwise rotation and ultimately a lower Harrod-neutral rate at point A.



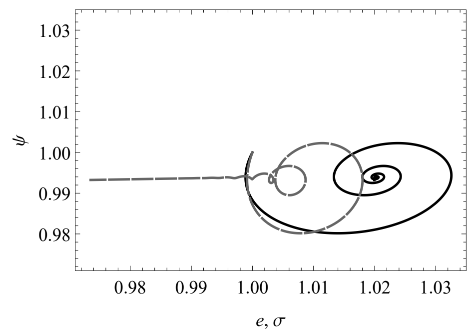
(a) KDC: Baseline



(b) CDC: Baseline



(c) KDC: z-shock



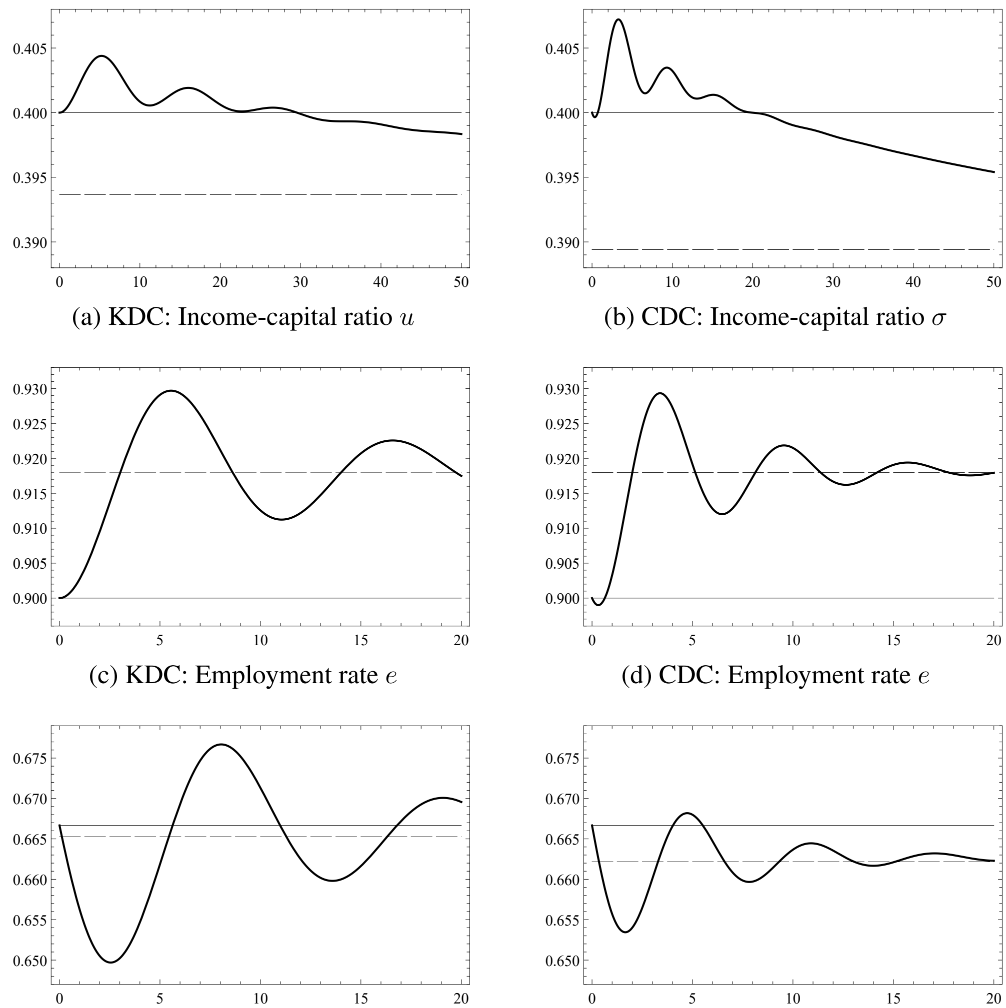
(d) CDC: z-shock

Figure 3: **Simulations: phase trajectories**. Each panel shows simulated phase trajectories, relative to baseline steady state values. Top row represents a baseline simulation, with employment rate e and income-capital ratio (σ or u) 2% above respective steady states on the horizontal axis, and the labor share ψ 2% below steady state on the vertical axis. Bottom row shows trajectories in response to a shock to z, with state variables in steady state at $t = 0$. In all these simulations, the trajectory of employment and labor share is plotted in black; while the trajectory of income-capital ratio and labor share is plotted in dashed gray. See Section 5 for discussion.

of panels, and CDC model output in the right column of panels.

The top row of Figure 3 illustrates convergence of both models from an initial condition out-of-steady-state. The panels show phase trajectories for (e, ψ) and $(\sigma U, \psi)$; to facilitate visualization, the variables are normalized by their respective steady states. The standard result of a counter-clockwise cycle in activity-labor share space obtains: activity variables (be that the employment rate or the income-capital ratio) lead the labor share.

The bottom row of this figure shows phase trajectories in response to the z-shock, assuming that the model is in steady state at time zero. Two observations stand out. First, the shock to z affects state variables in qualitatively the same manner across the two models. Both the income-capital ratio and the labor share fall, while the employment rate rises. Further, convergence to the new steady state also displays the Goodwin pattern, though the



(a) KDC: Income-capital ratio u

(b) CDC: Income-capital ratio σ

(c) KDC: Employment rate e

(d) CDC: Employment rate e

(e) KDC: Labor share ψ

(f) CDC: Labor share ψ

Figure 4: **Simulations: state variables**. Each panel shows simulated phase trajectories for the two model's state variables. Left column represents the KDC, the right column the CDC. The solid (dashed) line indicates baseline (post-shock) steady state. See Section 5 for discussion.

income-capital ratio's cyclical movement ultimately subsides.

The medium term trends of the simulated state variables' response to the decline in z are further visualized in Figure 4. The solid horizontal line indicates the initial (pre-shock) steady state, and the dashed horizontal line the steady state after the shock. The top row reports the income-capital ratio, which *rises* above the pre-shock steady state for a number of periods, before falling below that level on its path to converge to the new steady state. The labor share (in the third row) declines, and the employment rate (in the second row) rises. This pattern is a critically important facet of the theory laid out in this paper: macroeconomic activity as proxied by the income-capital ratio initially rises following an institutional shift adverse to labor, but ultimately declines towards a lower steady state.

The growth rates in Figure 5 provide further detail on this issue. The first row of panels is important in this regard, reporting the accumulation rate (or warranted growth rate). In both models, this rate initially rises above its pre-shock steady state (on average), before stagnating towards the lower post-shock steady state.

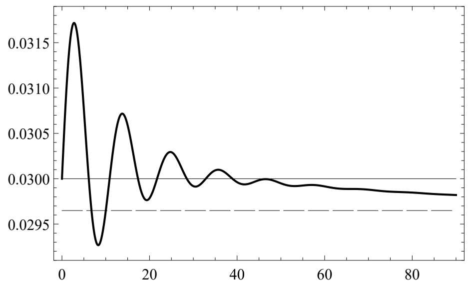
The remaining panels in rows 2-4 show the growth rates of output, labor productivity and capital productivity respectively. These shed light, in particular, on the effects of the z-shock *on impact*. For the left column of panels (describing the KDC trajectories), these growth rates all start at their pre-shock values. In contrast, the right column of panels (describing the CDC counterparts) shows a discrete change in these growth rates at $t = 0$. This is due to the fact that *instantaneous* firm optimization forces a jump to the new (rotated) IPF. Once this jump has occurred, movements along the IPF determine the path towards the new steady state.

Crucially, the pattern of technical change is Marx-biased on impact: the pre-shock iso-cost line is tangent to the IPF at the Harrod-neutral pre-shock rate of labor productivity growth. However, the clockwise rotation of the IPF implies that the post-shock tangency condition at the (as of yet) unchanged labor share is now in the North-West quadrant, where labor productivity is growing but capital productivity is declining.

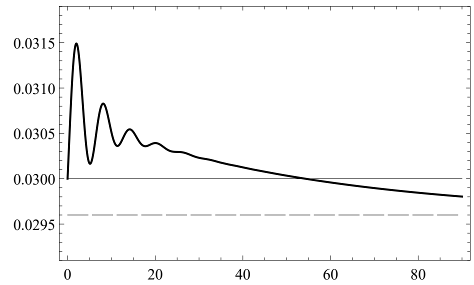
6 Conclusion

This paper has advanced two formal models building on Goodwin's seminal distributive cycle to argue that changing labor market institutions in the neoliberal era have played a key role in driving down the labor share, the income-capital ratio, and the growth rates of labor productivity and output. The income-capital ratio is endogenous in both models, but through different mechanisms. In the classical version, the main channel at play is the firm's choice of the direction of technical change; while in the Keynesian version, it is the principle of effective demand.

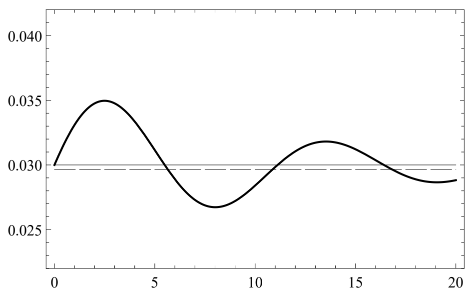
Both models conform to the empirical evidence at the business cycle frequency, i.e. show the Goodwin pattern; and conform to the stylized facts of the neoliberal era, and in particular a decline in the labor share of income coupled with economic stagnation.



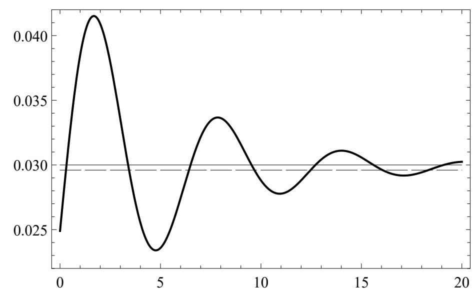
(a) KDC: Accumulation rate g



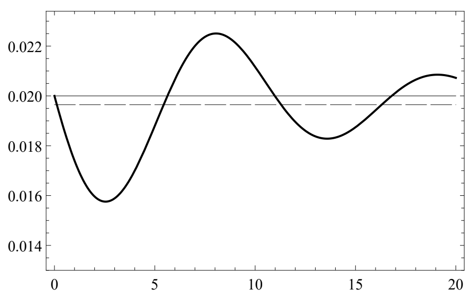
(b) CDC: Accumulation rate g



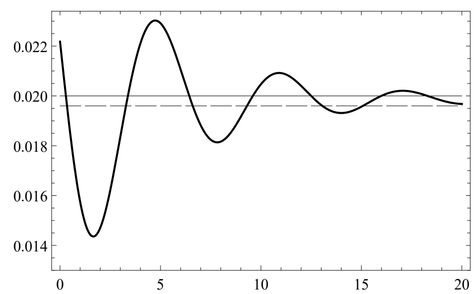
(c) KDC: Output growth h



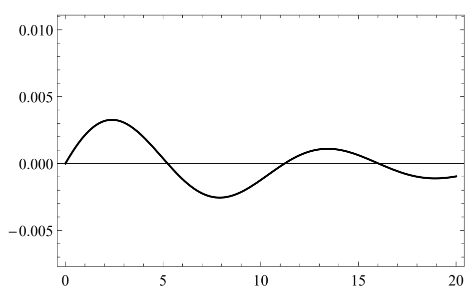
(d) CDC: Output growth $b + g$



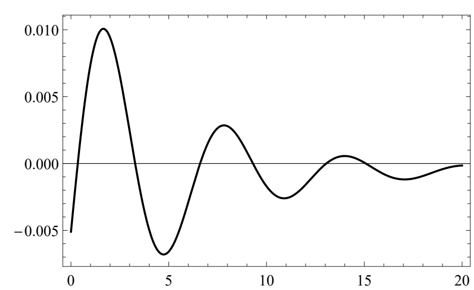
(e) KDC: Labor productivity growth a



(f) CDC: Labor productivity growth a



(g) KDC: Capital productivity growth b



(h) CDC: Capital productivity growth b

Figure 5: **Simulations: growth rates**. Each panel shows simulated phase trajectories for growth rates implicit in the two model's output. Left column represents the KDC, the right column the CDC. The solid (dashed) line indicates baseline (post-shock) steady state. See Section 5 for discussion.

Moreover, both models imply a potential tradeoff—that also appears to be borne out by US data in the relevant period—between labor-friendly policies aimed at reversing the rise in inequality and the long-run employment rate. In this respect, our frameworks are related to work by Flaschel and Luchtenberg (2013) on flexicurity: pro-labor (and pro-growth) policies also require compensatory measures to deal with their adverse effects in terms of long-run job destruction.

In conclusion, the two models differ significantly in the specific channels, but are fundamentally similar in their portrayal of the relevant short run and long run dynamics. The main goal of this paper was to draw out the specific assumptions necessary to arrive at such a point, and therefore to further debate both on theory and empirical applications.

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A Appendix

A.1 Local stability conditions

The Routh-Hurwitz conditions for local stability of three dimensional systems involve analyzing the Jacobian matrix J evaluated at the steady state. If we let $|J_{ij}|$ be the minor of J obtained by deleting the ith row and jth column, the conditions are:

$$\begin{aligned} Tr(J) &< 0 && \text{(A.1)}\\ |J_{11}| + |J_{22}| + |J_{33}| &> 0 && \text{(A.2)}\\ |J| &< 0 && \text{(A.3)}\\ -Tr(J)(|J_{11}| + |J_{22}| + |J_{33}|) + |J| &> 0. && \text{(A.4)} \end{aligned}$$

A.2 Keynesian model

We reproduce the Jacobian matrix (equation 3.10), and include the sign pattern implied by assumptions stated in the main text:

$$J^* = \begin{bmatrix} u(h_u - s_\pi(1-\psi)) & uh_e & u(h_\psi + s_\pi u) \\ eh_u & eh_e & e(h_\psi - a_\psi) \\ 0 & \psi\omega_e & -\psi a_\psi \end{bmatrix} \tag{A.5}$$

The corresponding sign pattern is then:

$$\begin{bmatrix} - & - & - \\ + & - & - \\ 0 & + & - \end{bmatrix}$$

Under these assumptions, inequalities A.1 and A.2 hold. In fact, $Tr(J) = u(h_u - s_\pi(1 - \psi)) + eh_e - \psi a_\psi < 0$; and $|J_{11}| + |J_{22}| + |J_{33}| > 0$ since $|J_{11}| = -e\psi(h_e a_\psi + \omega_e(h_\psi - a_\psi)) >$

0, $|J_{22}| = -u(h_u - s_\pi(1-\psi))\psi a_\psi > 0$, and $|J_{33}| = -ues_\pi(1-\psi)h_e > 0$. If j_{ij} is the ith row and jth column element of J, A.3 requires $j_{11}|J_{11}| - j_{21}|J_{21}| = j_{11}j_{22}j_{33} - j_{33}(j_{11}j_{22} - j_{21}j_{12}) + j_{21}j_{23}j_{13} < 0$. From the Jacobian, $j_{11}j_{22}j_{33} < 0$ and $j_{21}j_{23}j_{13} < 0$, while $-j_{33}(j_{11}j_{22} - j_{21}j_{12}) = ues(1-\psi)h_e < 0$. Hence, inequality A.3 holds.

Inequality A.4 is more difficult to ascertain, but a sufficient condition for it to hold is $-(eh_e - \psi a_\psi) > uh_u$: Rearranging gives $(j_{11} - Tr(J))|J_{11}| - j_{21}|J_{22}| - Tr(J)(|J_{22}| + |J_{33}|) > 0$, where $j_{11} - Tr(J) = -(j_{22} + j_{33}) = -Tr(J_{11})$. Substituting and distributing gives:

$$\underbrace{-Tr(J_{11})|J_{11}|}_{I}\underbrace{-j_{21}|J_{22}|}_{II}\underbrace{-Tr(J)|J_{22}|}_{III}\underbrace{-Tr(J)|J_{33}|}_{IV} > 0 \tag{A.6}$$

These terms I–IV can be signed, and a sufficient condition for A.4 to hold is $I + II > 0$. Rearranging gives:

$$\underbrace{[Tr(J_{11}) + uh_u]}_{+/-}\underbrace{h_e a_\psi}_{-} + \underbrace{\omega_e}_{+}\underbrace{[(Tr(J_{11}) + uh_u)}_{+/-}\underbrace{h_\psi}_{-} - \underbrace{Tr(J_{11})a_\psi}_{-} + \underbrace{uh_u s_\pi u}_{+}]$$

which is positive if

$$Tr(J_{11}) + uh_u < 0 \Leftrightarrow -(eh_e - \psi a_\psi) > uh_u. \tag{A.7}$$

While this is only sufficient, it is straightforwardly interpreted: the stabilizing elements along the trace have to outweigh the destabilizing element h_u. In particular, h_u appears in the law of motion of the employment rate, and there can lead to violation of the fourth Routh-Hurwitz inequality.[7]

A.3 Classical model

The Jacobian matrix of this system, evaluated at the steady state (without starring of variables, for brevity):

$$J = \begin{bmatrix} 0 & e(b_\psi - s\sigma - a_\psi) & es_\pi(1-\psi) \\ \psi\omega_e & -a_\psi & 0 \\ 0 & \sigma b_\psi & 0 \end{bmatrix} \tag{A.8}$$

[7]Note further that the fourth Ruth-Hurwitz inequality ensures that the real parts of a potential pair of complex eigenvalues are negative. Numerical simulations confirm that increases in h_u can lead to a Hopf bifurcation as the real parts pass through zero from below. A stable limit cycle emerges (with linear behavioral functions). Details are available upon request.

And its sign pattern is:

$$\begin{bmatrix} 0 & - & + \\ + & - & 0 \\ 0 & - & 0 \end{bmatrix}$$

Inequality A.1 is satisfied as $Tr(J) = -a_\psi < 0$. Remembering $b_\psi < 0$, A.3 is also verified since $|J| = e\sigma\psi s(1-\psi)\omega_e b_\psi < 0$. Furthermore, since $|J_{11}| + |J_{22}| = 0$, A.2 holds, too: $|J_{33}| = -e\psi\omega_e(b_\psi - s\sigma - a_\psi) > 0$. Similarly to the Keynesian model, (A.4) requires some more work. Remembering $a_\psi = f_{\hat{\sigma}} b_\psi$, we find $-|J_{33}| + |J|/Tr(J) = e\psi\omega_e(b_\psi - s\sigma - a_\psi) - e\sigma\psi s(1-\psi)\omega_e b_\psi / a_\psi = e\psi\omega_e\left(b_\psi - s\sigma - a_\psi - s\sigma(1-\psi)/f_{\hat{\sigma}}\right)$. Further, recall that $-f_{\hat{\sigma}} = \frac{1-\psi}{\psi}$, we find $-|J_{33}| + |J|/Tr(J) = e\psi\omega_e\left(b_\psi - s\sigma - a_\psi + s\sigma\psi\right)) = e\psi\omega_e\left(b_\psi - s\sigma(1-\psi) - a_\psi\right) < 0$.

A.3.1 Short-run adjustment of labor productivity growth

Let us prove that the optimal level of labor productivity growth is a negative function of z *on impact*. Start at $t = 0$ in steady state and consider a shock that takes the state of the labor market from z_0 to z_1, with $z_0 > z_1$. In steady state, labor productivity growth is $f(0; z_0) \equiv \bar{a}$. Define $\hat{\sigma}^\bullet$ as the level of capital productivity growth such that $f(\hat{\sigma}^\bullet; z_1) = f(0; z_0) = \bar{a}$. Notice that since f is increasing in z and decreasing in $\hat{\sigma}$, it follows that $\hat{\sigma}^\bullet < 0$. Let us now move to the slope of the f function in the two points $(\hat{\sigma}^\bullet, z_1)$ and $(0, z_0)$. Under the assumption $\frac{\partial f_{\hat{\sigma}}(0,z)}{\partial z} > 0$, at the given level of labor productivity growth $\bar{a}$ the after shock labor productivity growth is a steeper function of $\hat{\sigma}$ (in absolute terms); hence $f_{\hat{\sigma}}(\hat{\sigma}^\bullet; z_1) < f_{\hat{\sigma}}(0; z_0)$. Notice that using 4.2 the steady state equilibrium requires $f_{\hat{\sigma}}(0; z_0) = -\frac{1-\psi}{\psi}$. Define $\hat{\sigma}^{\bullet\bullet}$ as the after shock optimal level of capital productivity growth, that is $\hat{\sigma}^{\bullet\bullet}$ such that $f_{\hat{\sigma}}(\hat{\sigma}^{\bullet\bullet}; z_1) = f_{\hat{\sigma}}(0; z_0) = -\frac{1-\psi}{\psi}$. Since $f_{\hat{\sigma}\hat{\sigma}} < 0$, and $f_{\hat{\sigma}}(\hat{\sigma}^\bullet; z_1) < f_{\hat{\sigma}}(0; z_0) = -\frac{1-\psi}{\psi}$, then $\hat{\sigma}^{\bullet\bullet} < \hat{\sigma}^\bullet$. Finally, given $f_{\hat{\sigma}} < 0$, we have $f(\hat{\sigma}^{\bullet\bullet}; z_1) > f(\hat{\sigma}^\bullet; z_1) = f(0; z_0)$. The drop in z raises labor productivity growth.