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Household credit-financed consumption and the debt service ratio: tackling endogenous autonomous demand in the Supermultiplier model

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Household credit-financed consumption and the debt service ratio: tackling endogenous autonomous demand in the Supermultiplier model

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Abstract

The paper develops a Supermultiplier model where household debt-financed consumption is the autonomous component of demand driving growth. However, instead of taking autonomous consumption growth as exogenous - as usually done in canonical Supermultiplier models - we assume households' debt service ratio partially determines it. More precisely, we define a consumption function that captures: (i) the fact that households' demand for credit may depend on the burden interest payments have on their income (wages) and (ii) that credit conditions may also affect the pace of household expenditures. There are two equilibria in the model: one with a lower debt ratio and higher growth rate; and the other with a higher debt ratio and lower growth rate. Both equilibria are locally stable for the chosen set of parameters, yet the system converges to the steady state with a lower household debt ratio and higher growth rate.

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Both real and financial variables affect the steady growth path in the model, with the wage share and firms' propensity to invest having a positive effect on growth while the interest rate has a negative one.

Keywords: demand-led growth, Supermultiplier, household debt, consumption, endogenous autonomous demand

JEL classification codes: B50, C61, E11, G51, O41

1 Introduction

It is now widely documented that there has been a secular deterioration of income distribution in the US, as well as in other advanced economies (Atkinson et al., 2011; Piketty & Saez, 2003; van Treeck, 2014). It has also been argued that as income inequality worsened throughout the 20th century, household indebtedness increased so as to avoid consumption stagnation, especially in Anglo-Saxon economies (Kumhof et al., 2015; Kumhof et al., 2012; Rajan, 2010; van Treeck, 2014). As a consequence of these observations, more recently, the issue of household debt dynamics has been emphasized in both mainstream and heterodox macroeconomics. We can divide the heterodox literature into two main approaches: (i) demand-led growth theories that incorporate household debt dynamics into their model (e.g: Dutt, 2005, 2006; Fagundes, 2017; Hein, 2012a; Pariboni, 2016; Setterfield and Kim, 2017, 2020 among others); (ii) and heterodox models that have attempted to look at it from a stock-flow consistency perspective (e.g.: Byrialsen and Raza, 2020; Caverzasi and Godin, 2015; van Treeck, 2009, among others). Meanwhile the mainstream approach has mostly focused on explaining some of these patterns through a general equilibrium framework (e.g.: Mian et al., 2021; Mian et al., 2017).

In this paper, we focus on demand-led growth theories that have attempted to incorporate household debt-dynamics into their models. This literature can be further divided into the neo-Kaleckian approach, which assumes a fully endogenous consumption function (See

Dutt, 2005, 2006; Hein, 2012a; Setterfield and Kim, 2017, 2020), and the Supermultiplier model, which assumes workers' autonomous consumption to be exogenously given (see Fagundes, 2017; Pariboni, 2016). While in the neo-Kaleckian framework consumption has to be fully endogenous to income - even when it is credit-driven - because investment is the autonomous component of demand, this is not the case in the Supermultiplier model. As a matter of fact, the latter assumes that non-capacity creating autonomous components of demand, such as government expenditures, household consumption out of credit, or capitalists consumption, lead growth in the long run. It also generally assumes that the autonomous components of demand grow at an exogenously given rate (Allain, 2015; Freitas & Serrano, 2015; Lavoie, 2016).

As far as we know, there is very scant work on the Supermultiplier front that formally deals with "semi-autonomous" (Fiebiger & Lavoie, 2017) or endogenous autonomous expenditures. In Brochier and Macedo e Silva (2019) and Brochier (2020), household consumption out of wealth is the main component of autonomous demand, and since wealth is explained within the model, so is the growth rate of autonomous expenditures. Ferri and Tramontana (2020) consider that the pace of durable consumption is semi-autonomous since its growth rate depends partially on an exogenous component and partially on the unemployment rate. At last, Caminati and Sordi (2019) also build a Supermultiplier model where autonomous expenditures grow endogenously – in their case, R&D investment and capitalist consumption, that grows in line with productivity. Even if these works take very different routes to make autonomous expenditures endogenous, they have an essential feature in common: they all allow for analysing the determinants of the growth rate and the impact that factors such as income distribution may have on the latter.

We contribute to this literature following an alternative path to make autonomous expenditures endogenous. We explicitly tackle one of the possible determinants of household credit demand, making autonomous consumption out of credit partially endogenous. By doing that, we integrate part of the post-Keynesian literature concerned with household credit determinants (Dutt, 2006; Hein, 2012b; Setterfield & Kim, 2016) and the Supermultiplier literature that deals with household consumption as one of the possible growth drivers (Freitas & Serrano, 2015; Lavoie, 2016; Pariboni, 2016). The objectives of this exercise are as follow. First, we want to analyse the impact of real variables (e.g.: marginal propensity to invest and income distribution) and financial variables (e.g.: debt service) on growth when it is driven by credit-financed consumption. Secondly, we want to explore how credit and consumption can help us understand the relationship between income distribution and growth following a demand-led growth framework.

With that in mind, we build a Supermultiplier model where autonomous, but endogenous, workers' consumption is the component of demand that drives growth. More precisely, we develop a debt-driven growth model in which the debt service affects household credit demand and, therefore, household autonomous consumption. We solve the model for the steady state and find two equilibria: one with a high rate of output growth and a low debt to income ratio; and another one with a low rate of economic growth but high debt to income ratio - and we analyse their stability. Given the complexity of the model, we propose a numerical illustration in order to better understand its' dynamics. For the chosen set of parameters, we show that the high growth-low debt equilibrium is more stable. As a final exercise we look at the derivatives of the more stable steady state equilibrium. We find that income distribution has a positive effect on growth; while financial variables, such as interest rates or the sensitivity of households to their debt burden, have a negative effect.

Beyond this introduction, the paper is divided into five sections as follows. Section 2 presents the model and derives its short run equilibrium. Section 3 presents the model's steady state solution in its analytical format and also a discussion of its local stability. Section 4 presents a numerical illustration of the model in which we analyse both steady state solutions, as well as their stability, using a chosen set of values for the parameters. Section 5 discusses the relevant derivatives of the steady state equilibrium which was found to be more stable. Section 6 concludes.

2 A credit driven household consumption model

The model deals with a pure credit closed economy with no inflation and without government composed by three institutional sectors: households, banks and firms. Following the post-Keynesian literature (see Dutt, 2005, 2006; Hein, 2012b; Setterfield and Kim, 2016, 2017, 2020 among others), we further divide the household sector into two groups - workers and rentiers. Rentier households buy all the equities issued by firms, e, and hold the rest of their wealth as deposits, M_r, at banks. Worker households take on loans, L_w, to consume. Since workers do not consume all their after-interest wage income, they accumulate wealth in the form of deposits, M_w, at banks (the only asset they hold). Banks give out loans to worker households and take on household deposits. Firms issue equities to households to finance the part of investment not covered by retained earnings.

Rentier households consume (C_r) part of their financial income (accruing from dividends, FD and FB). Workers finance part of their consumption with credit and part with their wage income after interest on loans (C_w).[1] Banks distribute all of their profit, FB, to rentier households. Firms produce an homogeneous good that is used both for consumption, C, and investment, I, purposes. Tables 1 and 2 detail respectively the balance sheet and the transactions and flow of funds of the institutional sectors.

Table 1: Balance sheet matrix

	Households		Firms	Banks	Total
	Workers	Rentiers			
Deposits	$+M_w$	$+M_r$		$-M$	0
Equities		$+ep_e$	$-ep_e$		0
Loans	$-L_w$			$+L_w$	0
Firm's Capital			K		K
Total	V_w	V_r	$K - ep_e$	0	K

[1] See Appendix A for the model's full set of equations.

Table 2: Transaction flow matrix

	Households		Firms		Banks		Total
	Workers	Rentiers	Current	Capital	Current	Capital	
Consumption	$-C_w$	$-C_r$	$+C$				0
Firms' Investment			$+I$	$-I$			0
Wages	$+W$		$-W$				0
Firm's Profits		$+FD$	$-(FD+FU)$	$+FU$			0
Bank's Profits		$+FB$			$-FB$		0
Interest on Loans	$-i_l L_w$				$+i_l L_w$		0
Interest on Deposits	$+i_m M_w$	$+i_m M_r$			$-i_m M$		0
Change in Loans	$+\dot{L}_w$					$-\dot{L}_w$	0
Change in Deposits	$-\dot{M}_w$	$-\dot{M}_r$				$+\dot{M}$	0
Change in Equity issues		$-p_e\dot{e}$		$+p_e\dot{e}$			0
Total	0	0	0	0	0	0	0

In what follows, we describe the behaviour of all aggregate demand components, focusing on workers' credit financed consumption as it is the autonomous expenditure component that leads growth in the model.

2.1 Aggregate demand behaviour

2.1.1 Workers' consumption

Following the Supermultiplier literature, as in Fagundes (2017) and Pariboni (2016), we assume that workers take on new loans ($\dot{L}_w$) to finance autonomous consumption (Z) (equation 1). We also assume that this autonomous component of demand grows at a rate g_z (equation 2). For simplification purposes we abstract from household debt amortization.[2]

$$\dot{L}_w = Z \tag{1}$$

$$Z = Z_0 e^{g_z t} \tag{2}$$

[2] For Supermultiplier models that take household debt amortization into account see Fagundes, 2017; Pariboni, 2016.

However, instead of taking the growth of debt-financed consumption as exogenously given, we suppose that household consumption out of credit is partially endogenous to the model or semi-autonomous (Fiebiger & Lavoie, 2017).[3] More precisely, we assume it depends positively on autonomous factors (φ_0) and negatively on the debt service to workers' income ratio (ds):

$$g_z = \varphi_0 - \varphi_1 ds \tag{3}$$

Where φ_1 represents a measure of how sensitive household credit demand for consumption is to the debt service and also to the credit conditions as represented by the interest rate on loans. The debt service ratio is given by equation 4.

$$ds = \frac{i_l L_w}{W} = \frac{i_l L_w}{wY} \tag{4}$$

Where L_w is workers' accumulated debt, i_l is the interest rate on workers' loans, Y is total income and w is the wage-share. Substituting 4 into 3, we arrive at equation 5:

$$g_z = \varphi_0 - \varphi_1 \frac{i_l}{w} l_w \tag{5}$$

Where $l_w = \frac{L_w}{Y}$ is the total loans to output ratio.

This specification for the growth rate of household autonomous consumption builds on the work of Dutt (2006), Isaac and Kim (2013), and van Treeck (2009) among others, taking into account the fact that debt servicing (ds) may on average affect the pace of new credit demand for consumption purposes (g_z).

As for the debt burden, we write it as a ratio between interest payments on loans and wages (instead of disposable income) in a rough approximation to the Minskyan typology (see Cynamon and Fazzari (2008) on Minsky's typology applied to the household sector), where interests on loans represent workers' cash commitments and wages their cash flow. As highlighted by Setterfield and Kim, 2020, for indebted households the limit of Ponzi finance (as applied to firms) is not feasible since households would not be able to use all

[3]For more on the discussion of endogenous autonomous expenditures see Allain, 2021; Brochier and Macedo e Silva, 2019; Fazzari et al., 2020; Fiebiger and Lavoie, 2017.

their income to debt servicing and still provide for their subsistence. Therefore, we assume that on average the debt service ratio is positive but lower than one ($0 < ds < 1$).

Besides credit-financed autonomous consumption, workers also consume a fraction (α_1) of their disposable income (Y_{d_w}) (equation 6):

$$C_w = \alpha_1 Y_{dw} + Z \tag{6}$$

Where workers' disposable income is given by the sum of their earning from wages plus the difference between what they receive as interest on their deposits, $i_m M_w$, and what they must pay as interest on loans they take, $i_l L_w$ (equation 7):

$$Y_{dw} = W + i_m M_w - i_l L_w \tag{7}$$

In this first version of the model, we'll assume for simplification purposes that deposits earn no interest, $i_m = 0$. That means workers' disposable income will amount to wages net of interest payments on loans. It also means that only rentiers earn financial income, as deposits are the only asset of workers.

2.1.2 Rentiers' consumption

We assume that rentiers households earn financial income accruing from firms' distributed profits (FD), banks profits (FB) and interest payments on deposits (equation 8). Since we initially assume that banks pay no interest on deposits ($i_m = 0$), rentiers disposable income will amount to total distributed profits.

$$Y_{dr} = FD + FB + i_m M_r \tag{8}$$

We assume that firms retain a fraction (s_f) of their total profit (πY) and distribute the rest of it to rentiers (equation 9).

$$FD = (1 - s_f)\pi Y \tag{9}$$

In turn, banks profit from the interest differential of their assets and liabilities (equation 10) and distribute all of it to rentiers.

$$FB = i_l L_w - i_m M \tag{10}$$

In this version, as $i_m = 0$, banks' profits will be equivalent to interests charged on loans, $i_l L_w$.

At last, rentier households consume a fraction of their disposable financial income (equation 11).

$$C_r = \alpha_2 Y_{dr} \tag{11}$$

2.1.3 Firms' investment

As for firms' investment, we assume it to be endogenously determined by current income, Y, and firms' marginal propensity to invest, h (equation 12). We also suppose, as in Freitas and Serrano (2015), that firms' adjust their investment behavior to the discrepancies between the actual, u, and the normal, u_n, capacity utilization rate (equation 13). This provides a conditional solution to the problem of Harrodian instability as highlighted by Allain (2015). [4]

$$I = hY = \dot{K} \tag{12}$$

$$\dot{h} = h\gamma(u - u_n); \tag{13}$$

It follows from the assumptions in equations 12 and 13 that the rate of capital accumulation in this economy will be a function of the marginal propensity to invest and the rate of capacity utilization, both divided by the capital output ratio, v (equation 14):

$$g_K = \frac{hu}{v} \tag{14}$$

[4] See Hein (2014), Hein et al. (2012), and Skott (2010) for a review on this debate.

As should be clear from equations 12 and 14, we also abstract from capital depreciation.

2.2 Short-run equilibrium

By substituting the behavioural equations 6, 11 and 12 into the identity between supply and demand (equation 15) and solving for output, we obtain the short run goods market equilibrium level of income (equation 16).

$$Y = C_r + C_w + I \tag{15}$$

$$Y = \frac{Z + aL_w}{(s-h)} \tag{16}$$

In equation 16: (i) $a = -(\alpha_1 - \alpha_2)i_l$ represents the net effect on aggregate demand (and income) of the interest payment on loans; (ii) $s = 1-\alpha_1 w-\alpha_2 s_d \pi$ is the marginal propensity to save. [5]

Dividing both sides of equation 16 by the full capacity output level ($Y_{fc} = K/v$), we then find the short-run rate of capacity utilization, u:

$$u = \frac{v[(g_l + a)l_{k_w}]}{(s-h)} \tag{17}$$

Where (i) $g_l = \frac{\dot{L}_w}{L_w} = \frac{Z}{L_w}$ is the loans growth rate; (ii) $l_{k_w} = \frac{L_w}{K}$ is the worker's loans to capital ratio; and (iii) v is the capital-output ratio. It must be emphasized at this point that since both l_{k_w} and g_l have entered in the equation that describes the short run equilibrium for the rate of capacity utilization, they will both play a fundamental role in defining the long run steady state dynamics of the model. This will be seen in further details in subsection 3.1.

Assuming the Keynesian stability condition holds in the model ($s > h$), for having a positive capacity utilization rate, the numerator of equation 17 has to be positive. This will be the case when $g_l > -a$, that is, when the positive effect of autonomous consumption (new

[5] The details of this derivation can be found in Appendix B.

credit) exceeds the negative effect interest payments on loans have on aggregate demand (as they reduce workers' disposable income and, therefore, induced consumption).[6]

Since it will be instrumental for deriving the steady state solution for the model, we also present the short run autonomous consumption to output ratio (z) (equation 18). Equation 18 is obtained by dividing both sides of equation 16 by the level of output and solving it for the autonomous consumption ratio.

$$z = s - h - al_w \tag{18}$$

Where $z = \frac{Z}{Y}$ is the ratio of autonomous consumption (new loans) to income. Arriving at an analytical solution to the model will also require that we explicitly account fo the short run output growth of the economy, g_y (equation 19). We arrive at equation 19 by taking the total derivative of equation 16 and normalizing it by the level of output. In Appendix B, we provide the full derivation of the short run output growth rate.

$$g_Y = \frac{l_w}{(s-h)}\left[g_l(\varphi_0 - \varphi_1 \frac{i_l}{w} l_w + a) + \frac{h\gamma}{(s-h)}(g_l + a)\left(\frac{v(g_l + a)l_{k_w}}{(s-h)} - u_n\right)\right] \tag{19}$$

Having presented the short run capacity utilization rate and the key variables for analysing the model, in section 3 we present the main results of the steady state solution.

3 Steady state solution

In the previous section we defined the model and its key behavioural assumptions. Since workers' credit-financed consumption growth rate is partially determined by the debt to income ratio, l_w (see equation 5), the key variables and ratios of the model can also be written as functions of l_w. Consequently, taking into account that in steady growth we must have $g_k = g_y = g_l = g_z$ and $u = u_n$, we find that the steady state solution of the model

[6] See Brochier and Freitas (2022) on this.

is given by the following equation:

$$l_w^2 + bl_w + c = 0 \tag{20}$$

Where $b = \frac{v}{u_n} + \frac{w}{\varphi_1 i_l}[i_l(\alpha_1 - \alpha_2) - \varphi_0]$ and $c = \frac{w}{\varphi_1 i_l}(1 - \alpha_1 w - \alpha_2 s_d \pi - \frac{v}{u_n}\varphi_0)$. The full derivation of the steady state solution of the model can be found in Appendix C. As a result of equation 20, the steady state solution of the model is defined by two roots for the loans to income ratio:

$$l_w^* = -b \pm \sqrt{b^2 - 4c} \tag{21}$$

We have then arrived at two possible long run equilibria for the model fully determined by the parameters describing the internal dynamics of the model. In other words, the immediate consequence of having credit-financed consumption as the autonomous, but yet endogenous, component of demand that drives growth in a Supermultiplier model is two-fold: i) the steady state value of l_w is fully described by a combination of the model main parameters; ii) the steady state solutions will be fundamentally defined by the values of l_w as all of the other relevant variables will be explained by the dynamics of the debt to income ratio. More precisely for each l_w^* the full steady state solution of our model will be described by:

$$g^* = g_K^* = g_Z^* = \varphi_0 - \varphi_1 \frac{i_l}{w} l_w^* \tag{22}$$

$$h^* = \frac{g_Z^* v}{u_n} = \frac{v}{u_n}\left(\varphi_0 - \varphi_1 \frac{i_l}{w} l_w^*\right) \tag{23}$$

Since we have arrived at a two equilibria steady state solution, we must now look at the stability of the dynamical system to determine which equilibrium is stable. This is done in subsection 3.1.

3.1 Dynamic analysis of the steady state

The system describing the dynamics of the model is composed of four differential equations (equations 24). We start off with the fundamental dynamic equation of Supermultiplier models as described in equation 13. If we then replace the rate of capacity utilization by its short run result, as described in equation 17, we then get the first equation of our dynamical system (24). Since both g_l and l_{k_w} appear in this first equation we must also derive their dynamic behaviour. In order to do so, we depart from the very definition of the two variables and replace equation 14 (the rate of capital accumulation) in the equation for $\dot{l}_{k_w}$ and 5 (the rate of growth of autonomous consumption) in the equation for$\dot{g}_l$. The details of these derivations can be found in Appendix D. Finally, the fourth and last equation of the dynamical system is found by departing from the definition of l_w and replacing the rate of growth of output, g_y (equation 19).

$$\begin{array}{lcccl} \dot{h} & = & f[h, g_l, l_{k_w}] & = & h\gamma\left[\frac{v(g_l+a)l_{k_w}}{(s-h)} - u_n\right] \\ \dot{l}_{k_w} & = & f[h, g_l, l_{k_w}] & = & l_{k_w}\left[g_l - \frac{h(g_l+a)l_{k_w}}{(s-h)}\right] \\ \dot{g}_l & = & f[g_l, l_w] & = & g_l(\varphi_0 - \varphi_1 \frac{i_l}{w} l_w - g_l) \\ \dot{l}_w & = & f[h, g_l, l_{k_w}, l_w] & = & l_w\left\{g_l - \frac{l_w}{(s-h)}\left[g_l(\varphi_0 - \varphi_1 \frac{i_l}{w} l_w + a)\right.\right. \\ & & & & \left.\left. + \frac{h\gamma}{(s-h)}(g_l + a)\left(\frac{v(g_l+a)l_{k_w}}{(s-h)} - u_n\right)\right]\right\} \end{array} \tag{24}$$

The dynamics of both the marginal propensity to invest and of the loans to capital ratio depend on the same three variables: firms' propensity to invest, h; household debt growth rate, g_l; and household debt to capital ratio, l_w. As for the dynamics of household debt growth rate it will only be affected by itself and by the household debt to income ratio. At last, the dynamics of l_w is affected by the value of all of the four variables of the dynamical system.

The analysis of the dynamical system described in 24 should allows us to understand whether or not the model developed in this paper is dynamically stable, i.e.: converges to any of the steady state solutions; or unstable, if it does not converge to any of the two steady state solutions described in equation 21. However, given that this is a non-linear four-dimensional system, it is not possible to arrive at an analytical solution to this issue. Therefore, to address the system's stability we adopt two complementary procedures: the first one is to look at the Routh-Hurwitz determinants of the Jacobian matrix of the system; the second approach is to look at behavior of the dynamical system through a numerical simulation exercise.

In what follows, we then present the Jacobian matrix of the system and evaluate it at the two steady state solutions. This allows us to analyze the local stability conditions of the dynamical system around the two steady state positions using Routh-Hurwitz determinants.

3.2 Routh-Hurwitz Determinants of the Jacobian Matrix

Since this is a non-linear dynamic system, we look at the Jacobian matrix to study the asymptotical stability of the the two equilibrium solutions found in the previous section, using a linear approximation (Gandolfo, 2010, p.385). Additionally, since this is a four by four system it is not enough to look at the determinant and trace of the Jacobian, one must look at the Routh-Hurwitz determinants of the Jacobian matrix (Gandolfo, 2010, p.239, p.269). [7] To start this exercise we first define our Jacobian matrix to be given by:

$$\begin{bmatrix} h_h & h_k & h_g & h_w \\ k_h & k_k & k_g & k_w \\ g_h & g_k & g_g & g_w \\ w_h & w_k & w_g & w_w \end{bmatrix} \tag{25}$$

Where h_h is defined as the partial derivative of $\dot{h}$ with respect to h, h_k with respect to l_{k_w}, h_g with respect to g_l and h_w with respect to l_w. k_x, g_x and w_x represent the partial derivatives of $\dot{l}_{k_w}$, $\dot{g}_l$ and $\dot{l}_w$, respectively. In appendix E, we show how the Jacobian matrix above is associated with the following characteristic polynomial:

$$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 \tag{26}$$

[7] See also Hein and Woodgate (2021), Nikolaidi (2014), and Spinola (2020) for other applications of this.

Where: $a_0 = 1$ and $a_1 = -(w_w + g_g + h_h + k_k)$. The definitions of the remaining coefficients can be found at the end of Appendix E. Our Routh-Hurwitz matrix then becomes:

$$\begin{bmatrix} a_1 & a_3 & 0 & 0 \\ a_0 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & a_0 & a_2 & a_4 \end{bmatrix} \tag{27}$$

And therefore, the conditions for local stability around our two steady state equilibria are given by: $\Delta_1 = a_1 > 0$; $\Delta_2 = a_2 a_1 - a_0 a_3 > 0$; $\Delta_3 = a_3 \Delta_2 - a_1(a_1 a_4) > 0$ and $\Delta_4 = a_4 \Delta_3 > 0$ (Gandolfo, 2010, p.239, p.269).The details of the derivation of our Routh-Hurwitz determinants, as well as the derivations for the Jacobian evaluated at the steady state can be found in Appendix E and F, respectively. However, once again, the complexity of the analytical solution of our model does not allow us to provide an economic interpretation of our results, with the only exception of the first condition stated above. It is easy to show that the first condition on the determinants of Routh-Hurwitz can easily be translated into a negative trace of the original Jacobian matrix as presented in equation 25.

From the definition of the Jacobian matrix, as well as the defining equations of the model we have that the trace of the Jacobian evaluated at the steady state position is given by:

$$T(J^*) = h_h^* + k_k^* + g_g^* + w_w^* \tag{28}$$

Where the partial derivatives evaluated at the steady state position can be shown to be[8]:

$$\begin{aligned} h_h^* &= \tfrac{\partial \dot{h}}{\partial h} = \tfrac{\gamma u_n h^*}{(s-h^*)} \\ k_k^* &= \tfrac{\partial \dot{l}_{k_w}}{\partial l_{k_w}} = -g_z^* \\ g_g^* &= \tfrac{\partial \dot{g}_l}{\partial g_l} = -g_z^* \\ w_w^* &= \tfrac{\partial \dot{l_w}}{\partial l_w} = -g_z^* \left[\tfrac{l_w^*}{(s-h^*)} (\varphi_0 + a) \right] \end{aligned} \tag{29}$$

[8]the details of this derivation can be found in Appendix F

Replacing each partial derivative evaluated at the steady state in equation 28 and making some mathematical manipulation, we get condition (30).

$$T(J^*) = \frac{\gamma u_n h^*}{(s-h^*)} - g_z^*\left[\frac{l_w^*}{(s-h^*)}(\varphi_0 + a)\right] - 2g_z^* \tag{30}$$

Where we can further replace both g_z^* and h^* by their steady state values, as described in equations 22 and 23, respectively, and analyse the sufficient condition for the trace to be negative given by the inequality (31). More precisely, given the trace of the Jacobian evaluated at the steady state (30), if we assume that $g_z^* > 0$ and $\varphi_0 + a + \frac{2v\varphi_1 i}{u_n w} > 0$, then for $T(J^*) < 0$ we must have, as a sufficient condition, that [9]:

$$\frac{\varphi_0 w}{\varphi_1 i_l} > l_w^* > \frac{\gamma v - 2\left(s - \frac{v\varphi_0}{u_n}\right)}{\varphi_0 + a + 2\frac{v\varphi_1 i_l}{u_n w}} \tag{31}$$

As expected, one of the necessary conditions for the local stability of the steady state results in constraints on the debt to income ratio evaluated at the steady state l_w^*.

A few parallels can be drawn with some of the results found in Hein and Woodgate (2021). First, just as they have found an upper and lower limit for g_z^* under a Supermultiplier model, here we have found an upper and lower limit to the debt-to-income ratio, which is directly related to limits on the autonomous demand component growth rate that is partially endogenous in this model. More precisely, the lower limit for the steady state value of the debt to income ratio can be translated to the following upper limit for the steady state rate of growth:

$$g_z^* < \frac{2(s - \frac{v\varphi_0}{u_n}) - \gamma v}{\varphi_0 + a + \frac{2v\varphi_1 i}{u_n w}} - \varphi_0 \tag{32}$$

Secondly, we notice from this condition that the higher the steady state rate of growth, the lower the harrodian instability parameter will have to be in order to ensure the stability of the system, cet. par.. Differently from the results of Hein and Woodgate (2021), the

[9]The details of this derivation can be found in Appendix G.

upper bound to the autonomous demand growth rate depends not only on real variables but also on financial ones, such as the interest rate.

Knowing that this is a necessary, but not sufficient, condition for local stability of the steady state solutions of the model, it is essential to further develop the analysis of the model's properties by means of numerical simulation.

4 Steady state analysis: numerical exercise

The numerical exercise proposed in this section aims at investigating the behavior of the model's dynamical system to understand (i) whether the system converges to one of the two equilibria of the model; (ii) whether the two equilibria - or one or none – are locally stable for a reasonable set of parameters by presenting the numerical values of the trace and determinant of both of them. The approach adopted in this section is highly inspired by the one of Ferri and Tramontana, 2020.

To calibrate the model, we estimate most of the relevant parameters and when available we rely on the estimations of previous works as presented in table 3. All parameters are estimated for the US economy. The latter presents a good case for illustration of the model's properties since it has been widely argued – see Barba and Pivetti (2009) and Cynamon and Fazzari (2008), for example – that credit-financed consumption has played an important role in boosting US recent economic performance, but also contributed to its financial instability. In table 4 we present the steady state results for all of the relevant variables assuming the parameters values presented on table 3.

Table 3: Parameter Values

Variables	Values (US Economy)	Source
α_1	0.9	Bunting, 1998 and Setterfield and Kim, 2020
α_2	0.2	Author's calculation[a]
u_n	0.79	Author's calculation[b]
i_l	0.02	Author's calculation[c]
π	0.4	Author's calculation[d]
v	3	Author's calculations[e]
φ_0	0.084	Author's calculation to guarantee $g_z > 0$
φ_1	0.06	van Treeck, 2009[f]
s_f	0.7	Skott and Ryoo, 2008
γ	0.02	Author's calculation[g]

[a] Author's estimation based on Ferri and Tramontana, 2020 and Fazzari et al., 2020;
[b] Calculated as the average of TCU data series from FRED;
[c] Calculated as the average of INTDSRUSM193N minus FPCPITOTLZGUSA data series from FRED;
[d] Calculated as the average of the complement of LABSHPUSA156NRUG data series from FRED;
[e] Calculated as the average of the ratio RKNANPUSA666NRUG to GDPCA data series from FRED;
[f] On Page 478, van Treeck, 2009 suggests a $b_1 = 0.05$ for one of his case analysis;
[g] Estimated to guarantee Harrodian stability following Fazzari et al., 2020;

At this point a few things are worth pointing out on the table 3. First, most of the values used in this part of the paper have been chosen based on empirical observations using data provided by the Federal Reserve database. This is the case for u_n, the normal rate of capacity utilization; i_l, that we chose based on an estimate of the real interest rate (discounted for inflation); and π, the profit share. One can notice the values we have found seem to be in accordance with what is commonly adopted in the literature.[10] The only exception is the value for the capital-output ratio that is considerably high, but that still falls within a range of economically meaningful values.

It is important to mention that v was the adjustment variable for calibrating the model's

[10] See for instance Haluska et al. (2021) and Gahn (2020) and their estimation of the normal rate of capacity utilization.

steady state. That means we have adopted the estimated values for the other parameters and controlled for a steady growth rate of the economy and household debt to income ratio that would make economic sense for the US economy, respectively $g^* < 10\%$ and $l_w^* < 250\%$. The value assumed by the capital-output ratio, therefore, has an impact on the steady state values of the relevant variables, although it does not significantly changes the stability of the system, as the model remains stable for a great range of meaningful values of v, other things equal.

Secondly, the parameters of the consumption function were taken from the current literature on demand-led growth models. Following Setterfield and Kim, 2020, we used the consumption and savings parameters as suggested by other authors, which can be found on table 3. Finally, we have chosen a value for γ that guarantees steady state stability. As has been pointed out by a few authors (Allain, 2015; Fazzari et al., 2020; Ferri & Tramontana, 2020; Hein & Woodgate, 2021; Lavoie, 2016) the Supermultiplier model needs a low γ in order to have stability. The results of this numerical exercise are presented in subsection 4.1.

4.1 Results of the numerical exercise

Table 4: Steady State Results

Variables	Steady state 1	Steady state 2
l_w^*	2.00372	29.1988
g^*	0.0799926	0.0256024
$l_{k_w}^*$	0.527647	7.68902
h^*	0.303769	0.0972242
Local stability analysis (R-H Determinants)		
$\Delta_1(J^*)$	0.186223	0.172863
$\Delta_2(J^*)$	0.00177806	0.00109485
$\Delta_3(J^*)$	3.08×10^{-7}	8.71×10^{-8}
$\Delta_4(J^*)$	9.99×10^{-15}	3.85×10^{-15}
Constraints for stability		
C_1	42	
C_2	30	
C_3	-2.92088	

In table 4 we observe that the steady state equilibrium solution for l_w^* (21) results in two positive real roots. More precisely, there is one steady state position characterized by a low debt ratio, l_w^*, but a higher rate of growth of the economy, g^*, (Steady State 1); and the other one by a high debt ratio and lower growth rate (Steady State 2). Steady State 1 arrives at more reasonable results for the values of growth and the debt to income ratio combined. It still reports steady state values for the marginal propensity to invest, h, and the debt to capital ratio, l_{k_w} higher than those empirically observed. Since this is a very simple model, without government spending or an external sector, we only have two components of aggregate demand, consumption and investment. That means if we assume a realistic share of household consumption to output, the remaining component of demand (investment) will comprehend the demand of the rest of the economy that is not explicitly modeled. The same thing can be said for the debt ratio. Since only working households take out loans, then all of the debt that happens in this economy is falling under workers' debt and that is the reason why we find such high values for l_w^*.

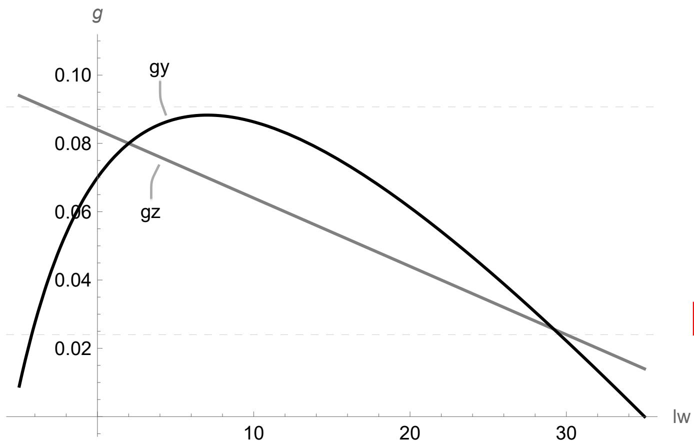
Secondly, table 4 also presents the numerical value of the sufficient condition for the trace to be lower than zero, which implies a range of the values for l_w^* (see equation 31). C_1 is the upper bound that this condition imposes on household debt to income ratio, and C_3, is the lower bound. The numerical values of the two conditions show that both steady state values of l_w^* satisfy the required condition for local stability of a negative trace. Furthermore, since we have positive values for all of the Rouht-Hurwitz determinants of the Jacobian for both steady state positions, we then find that both steady state solutions satisfy the required conditions for local stability.

At last, following the work of Setterfield and Kim, 2020, we asked ourselves if the two steady state positions are actually compatible with households being able to service their debt and still keep some positive consumption out of wages. In other words, if the two steady state positions were financially sustainable from the perspective of households. This constraint on l_w^* was derived into condition C_2, that represents the limit of financial sustainability,[11] and is also presented in table 4. We actually find that both results are financially sustainable from the perspective of households, yet steady state 2 is very close to the upper debt to income ratio beyond which workers' financial situation would not be sustainable. The two steady state solutions and their relationship towards the above mentioned constraints are illustrated in figure 1.

[11] Condition 2 was arrived at by assuming consumption out of wages net of interest payments is positive: $\alpha_1(W - i_l L_w) > 0$, which implies that $l_w^* < \frac{w}{i_l}$.

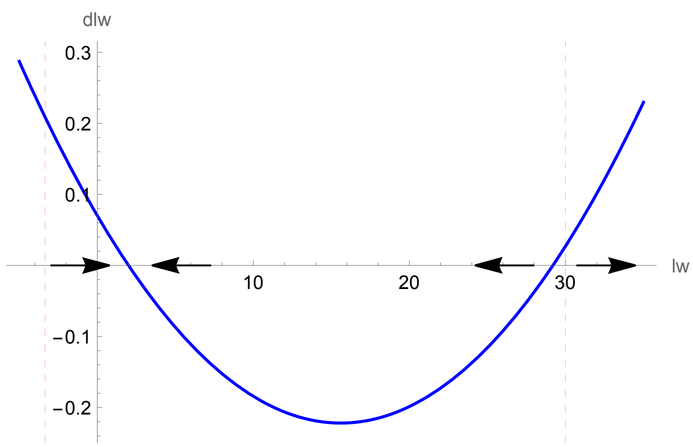
Figure 1: The two equilibria

(a) Growth rates



In (a) the grey line represents equation 3 and the black line represents equation 19. Both evaluated for the steady state parameters of table 3. The grey dashed lines represent the growth rates associated with the debt ratio limits of conditions C_2 and C_3.

(b) Household debt ratio



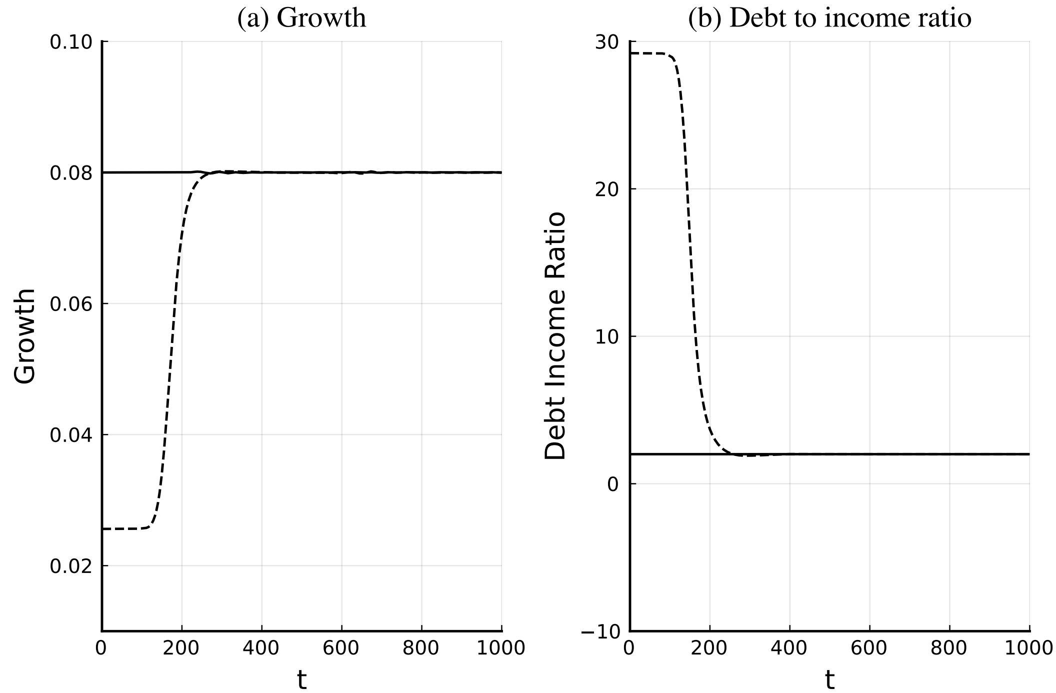
In (b) the blue line represents equation 21 evaluated for a range of household debt ratio. The red dashed lines represent conditions C_2 and C_3.

For the steady state parameters of table 3, we can draw graphs (a) and (b) of figure 1. In graph 1 (a), the grey line represents the growth rate of autonomous expenditures while the black line represents the output growth rate evaluated for the steady state parameters for a wide range of debt to income ratio values. The two equilibria of table 4 are represented by the intersection points of both curves. In graph 1 (b), the blue line shows the values assumed by equation 54 for a range of household debt ratio. The two steady state debt to income ratio roots are shown by the points where the curve intercept the x-axis. From graphs 1 (a) and (b), one can also notice that the two equilibria fall in between the range of debt to income ratios given by the feasibility condition (C_2) and by the lower bound of the negative Trace sufficient condition (C_3). Despite both equilibria being feasible, for the parameters of table 3, only steady state 1 seems to be economically meaningful, as the household debt ratio associated with steady state 2 is extremely high. As a final illustration exercise of the two steady state equilibria, we decided to further look into the stability of each equilibrium to investigate if we could find one equilibrium that was more stable than the other. This final exercise is presented in subsection 4.2.

4.2 Reassessing the steady state stability of our model

Since we seem to have found two steady state results with local stability for both of them, we decided to further investigate if it would be possible to say which steady state solution is more stable, following the work of Ferri and Tramontana, 2020. With that in mind, the two graphs below present a simulation of the dynamical system described in equation 24 starting at both steady state positions presented in table 4.

Figure 2: Steady state stability



(a) Growth and (b) Debt to income ratio results for 1000 iterations of the dynamical system starting at steady state 1 (full line) and steady state 2 (dashed line)

In figure 2 we look at the results for the debt growth rate, g_l, on graph (a), and the debt to income ratio, l_w, on graph (b). The full black lines on both graphs represent what happens to the variable after 1000 iterations of Euler's approximation for the dynamical system described by equation 24 starting from the first steady state position (steady state 1). Meanwhile the dashed lines represent what happens to the variable after the same 1000 iterations of the same dynamical system, but now starting from the second steady state position (steady state 2). As we can see both dynamics seem to converge to a steady state position in which the debt growth rate is around 8% and the debt to income ratio is around 2 (200%), which were the values estimated for steady state 1. This result indicate that, in fact, steady state 1 seems to be more stable, since when the simulation starts off from steady state position 2 (dashed line) it seems to diverge from it and actually move towards values of g_l and l_w that are closer to the first steady state position.

This illustration exercise has shown that it is possible to find two steady state equilibria for the model described in section 2 that have an economic meaning. Furthermore, we have also shown that both steady state solution are consistent with local stability, but that one of them (steady state 1) seems to be more stable than the other. In section 5 we further explore steady state 1 by looking at the derivative of this steady state solution with respect to some economically significant parameters; i) wage share (w); ii) interest rate (i_l); iii) exogenous autonomos consumption (φ_0); and iv) workers' sensitivity to their debt burden (φ_1).

5 Effects of the parameters on the high growth-low debt steady state

Now that we have analysed the stability conditions of the system and have shown that, for the chosen set of parameters, it converges to the equilibrium with a higher growth rate and lower household debt to income ratio, we can focus on the effects of the core parameters on this steady state growth path. Since the derivatives are very complex, making it difficult to apprehend their economic reasoning analytically, we solve the derivatives numerically for the steady state parameter values of table 3.

Table 5: Effects of the parameters on the steady growth path

Parameters	Variables		
	l_w^*	g^*	h^*
φ_0	−106.618	1.213	4.611
φ_1	7.108	−0.081	−0.307
w	−0.711	0.008	0.031
i_l	21.323	−0.243	−0.922

Numbers show the partial derivatives of the main variables of the model in the steady growth path (steady state 1) with respect to the parameters presented in the first column.
The values for the parameters are the same employed in section 4 to illustrate the two equilibria.

The main variables of interest are the long run growth rate, firms' propensity to invest and household debt to income ratio. Therefore, we focus on the effects the key parameters have on these variables. In turn, the key parameters are the wage share, the interest rate, the exogenous component of household autonomous consumption growth rate and the sensitivity of household autonomous consumption growth rate to the debt service ratio. In table 5, we present the values of the derivatives for the equilibrium values of household debt ratio (second column), the growth rate (third column) and firms' propensity to invest (fourth column) with respect to the four key parameters (first column).

As expected in the Supermultiplier model, the exogenous component of autonomous consumption growth rate has a negative effect on household debt ratio in the long run. This happens since, cet. par., a faster pace of autonomous consumption will increase capacity utilization in the short run, triggering firms reaction to adjust capacity to demand. As capital and output grow temporarily at a faster pace than household credit demand, household debt ratio will decrease in the long run. Therefore, we notice that the paradox of debt in the long run is a result of the model, as in canonical versions of the Supermultiplier model that assume there is only one source of non-capacity creating autonomous expenditures (Allain, 2015; Freitas & Serrano, 2015; Lavoie, 2016).[12]. However, we are aware that this is not a necessary result of the Supermultiplier closure, as when there is more than one source of autonomous injections, a faster pace of autonomous spending in one institutional sector will be associated with a higher debt ratio for the respective sector (Freitas & Christianes, 2020).

We notice that an increase in households' sensitivity to credit conditions contributes to a higher household debt ratio in the long run. This happens since an increase in households' sensitivity to credit conditions would have a negative effect on the pace of households' credit demand and, therefore, on the level of activity. As the slower consumption out of credit contributes to reduce the capacity utilization rate and firms adjust their capacity utilization to the (presumably permanent) lower demand, the output growth rate will fall to a larger extent than the initial fall in the autonomous demand growth rate. Therefore, a higher (lower) sensitivity parameter will lead to a higher (lower) household debt ratio in the long run.

An increase in the wage share reduces the household debt ratio in the long run, as it both alleviates households' debt burden and further stimulates demand through induced consumption. At last, a higher interest rate on loans will lead to a higher household debt ratio in the long run as it increases households' debt burden and puts a drag on aggregate demand, since the net effect on households' consumption will be negative as long as the propensity to consume out of wages is higher than the one out of financial income.

As for the long run growth rate, the exogenous component of the credit-financed consumption growth rate will have a positive effect on growth since it represents precisely the autonomous factors that may fasten the pace of household credit demand. As for the sensitivity of growth to credit conditions, the higher households sensitivity to the average debt burden the lower the long run growth rate. As for the wage share, it has a mild positive effect on the growth rate, as it stimulates induced consumption and, therefore, may lead firms to perceive the higher demand as permanent, temporarily fastening the pace of accumulation. It also directly reduces workers' debt burden by increasing their share in

[12] For more on this see Brochier and Freitas (2022)

total income. A higher interest rate unequivocally reduces the growth rate in the long run as it increases the debt burden for any household leverage ratio but also contributes to a higher household debt ratio in the long run.

As for firms' investment behaviour, a faster pace of credit-financed consumption arising from autonomous and exogenous demand would lead to a higher propensity to invest in the long run. This happens since firms' will adjust their capacity utilization to the desired one, which requires a permanently higher investment rate as the trend growth rate is also higher. In turn, a higher sensitivity of autonomous consumption growth to credit conditions would have the opposite effect on firms' propensity to invest: it reduces the pace of credit-financed consumption and the output growth rate, and as firms react to the lower trend of demand, the propensity to invest decreases. A higher wage share reduces worker households' debt burden and, therefore, contributes to a faster pace of credit-financed consumption in the long run. Once again, this requires a higher propensity to invest as firms try to keep the capacity utilization around the desired level. At last, the interest rate will have a negative effect on firms' propensity to invest since it reduces the growth rate of credit-financed consumption and, thus, firms adjust their investment behavior downwards keeping capacity utilization compatible with the lower expected growth rate of sales.

This exercise points out to some important features of the model that we highlight in what follows. First of all, income distribution may have permanent growth effects in the supermuliplier model. Contrariwise to what has been argued in recent criticism to the Supermultiplier model (Nikiforos, 2018), income distribution may affect the trend growth rate, not only the average growth rate. This was already pointed out in Brochier and Macedo e Silva (2019). This is the case when the growth rate of autonomous expenditures is assumed to be partially endogenous to the model or "semi-autonomous" as coined by Fiebiger and Lavoie (2017). Secondly, financial variables affect real spending decisions in our Supermultiplier model. This is captured in the model by the debt service impact on the pace of worker households' credit demand for consumption. A worsening in credit conditions as represented by an increase in the interest rate and stronger reaction of credit demand for consumption to the burden of debt could lead to lower demand and growth initially through consumption and, in a second moment, induce firms to cut back on investment as they expect a sluggish level of activity, reducing their investment ratio.

6 Conclusion

Supermultiplier growth models usually present the autonomous component of demand that drives growth to be exogenously determined. Most certainly for simplification purposes. That said, it seems relevant to further explore within the model the determinants of autonomous demand, so as to understand how they affect growth. With that in mind, we presented a Supermultiplier growth model where household credit-financed consumption is the autonomous component of demand that drives growth. But instead of assuming an exogenous growth rate for this component, we assume it to be partially determined by the financial burden that loans may impose on household consumption.

We have solved the model analytically and found two steady state equilibria. From the steady state solution, we are able to derive an important feature of the model: in steady state, all of the relevant variables - the growth rate and the marginal propensity to invest - become a function of the debt to income ratio. Therefore, understanding what happens to this ratio is fundamental to understanding the long run dynamics of the model.

However, due to the complex nature of the model's analytical solution, further clarification of the model's properties – such as the stability conditions and the economic interpretation of the two equilibria – required a numerical illustration. From this numerical exercise, we found that both steady state solutions of the model are compatible with local stability. Yet, the steady state with higher growth and lower debt ratio (steady state 1) appears to be more stable. The low growth-high debt ratio steady state (steady state 2) , besides seemingly less stable, also presents weaker economic meaning for the chosen set of parameters, as the household debt ratio is far beyond what is observable in real economies.

We also looked at the relevant derivatives of the higher growth-lower debt ratio steady state (steady state 1). We emphasize two of this final illustration exercise results: (i) income distribution may have a permanent effect on the long run growth rate, as an increase in the wage share reduces the debt to income ratio, increasing the economy's growth rate; (ii) financial variables – such as interest rates and the sensitivity of workers to their debt burden – affect real steady state variables. More precisely, we have that an increase in interest rates or in households sensitivity to their debt burden decreases the growth rate, as well as the investment to output ratio in the long run.

These latter results show that the Supermultiplier model developed is able to account for permanent effects of both real and final variables on the long run growth rate. This was made possible by combining a demand-led growth model where household credit-financed consumption is the autonomous spending that drives growth, but it is not exogenous, as it is partially explained by households' reaction to their debt burden. Even though this simple model has already produced interesting insights and helped to clarify important features that are due to the modelling of semi-autonomous expenditures, we must keep in mind its very high level of abstraction.

For representing real economies experiences in a meaningful way it would require a much higher level of real and financial complexity. As a next step in that direction, in future research we intend to assume that other sectors - e.g.: firms or rentiers - take out loans and to incorporate other institutional sectors, such as government, and their (autonomous) spending behaviour.

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Appendix A - Defining Equations

Worker households' consumption function

$$C_w = \alpha_1 Y_{dw} + Z \tag{6}$$

$$Y_{dw} = W + i_m M_w - i_l L_w \tag{7}$$

$$\dot{L}_w = Z \tag{1}$$

$$Z = Z_0 e^{g_z} \tag{2}$$

$$g_z = \varphi_0 - \varphi_1 ds \tag{3}$$

$$V_w = M_w - L_w \tag{33}$$

Rentier households' consumption function

$$C_r = \alpha_2 Y_{dr} \tag{11}$$

$$Y_{dr} = FD + FB + i_m M_r \tag{18}$$

$$S_r = Y_{dr} - C_r \tag{34}$$

$$V_r = S_r + e\dot{p}_e \tag{35}$$

$$p_e = \frac{\lambda V_r}{e} \tag{36}$$

$$\lambda = \lambda_0 - i_m \tag{37}$$

Firms equations

$$I = hY = \dot{K} \tag{12}$$

$$\dot{h} = h\gamma(u - u_n); \tag{13}$$

$$p_e \dot{e} = I - FU \tag{38}$$

$$FU = s_f \pi Y \tag{39}$$

$$FD = (1 - s_f)\pi Y \tag{9}$$

Banks: borrowing decisions

$$i_l = (1 + \rho_b)i_m \tag{40}$$

Additionally, we assume, for simplicity of the analysis, that $i_m = 0$ and that the interest rate on loans, i_l is determined as a mark up, ρ_b over the central bank's interest rate, i_{CB}, such that:

$$i_l = (1 + \rho_b)i_{CB} \tag{41}$$

$$M = L_w \tag{42}$$

$$FB = i_l L_w - i_m M \tag{10}$$

Appendix B - Short-run solution

$$Y = C_r + C_w + I \tag{43}$$

$$Y = \alpha_1(W + i_m M_w - i_l L_w) + Z + \alpha_2[(1 - s_f)\pi Y + i_l L_w - i_m M + i_m M_r] + hY \tag{44}$$

Assuming that $i_m = 0$, $s_d = (1 - s_f)$ and $w = (1 - \pi)$:

$$Y = \alpha_1(wY - i_l L_w) + Z + \alpha_2(s_d \pi Y + i_l L_w) + hY \tag{45}$$

And further defining $s = 1 - \alpha_1 w - \alpha_2 s_d \pi$ and $a = -(\alpha_1 - \alpha_2)i_l$:

$$Y(s - h) = aL_w + Z \tag{46}$$

Deriving everything with respect to time we get:

$$\dot{Y} = \frac{\dot{Z}}{(s - h)} + \frac{a}{(s - h)}\dot{L}_w + \frac{\dot{h}}{(s - h)^2}(Z + aL_w) \tag{47}$$

$$g_Y = \frac{\dot{Y}}{Y} = \frac{\dot{Z}}{Z}\frac{Z}{Y}\frac{1}{(s-h)} + \frac{a}{(s-h)}\frac{\dot{L}_w}{Y} + \frac{\dot{h}}{Y(s-h)^2}(Z + aL_w) \tag{48}$$

$$g_Y = g_z\frac{z}{(s-h)} + \frac{az}{(s-h)} + \frac{\dot{h}}{(s-h)^2}(z + al_w) \tag{49}$$

Replacing equations $g_z = \varphi_0 - \varphi_1\frac{i_l}{w}l_w$, and $\dot{h} = h\gamma[u - u_n]$ in the equation above, we then get that:

$$g_Y = (\varphi_0 - \varphi_1\frac{i_l}{w}l_w)\frac{z}{(s-h)} + \frac{az}{(s-h)} + \frac{h\gamma}{(s-h)^2}(z + al_w)(u - u_n) \tag{50}$$

Finally, replacing $z = g_l l_w$ and $u = \frac{v[g_l - (\alpha_1 - \alpha_3)i_l]l_{k_w}}{(s-h)}$ in the equation above we get:

$$g_Y = \frac{l_w}{(s-h)}\left[g_l(\varphi_0 - \varphi_1\frac{i_l}{w}l_w + a) + \frac{h\gamma}{(s-h)}(g_l + a)(u - u_n)\right] \tag{51}$$

$$g_Y = \frac{l_w}{(s-h)}\left[g_l(\varphi_0 - \varphi_1\frac{i_l}{w}l_w + a) + \frac{h\gamma}{(s-h)}(g_l + a)(\frac{v[g_l - (\alpha_1 - \alpha_3)i_l]l_{k_w}}{(s-h)} - u_n)\right] \tag{52}$$

Appendix C - Steady State Solution

Since under steady state we must have $g_l = g_z$ this then results in:

$$\frac{z}{l_w} = \varphi_0 - \varphi_1\frac{i_l}{l_w}l_w \tag{53}$$

If we then assume that under steady state $z = z^*$ and solve the equation above for l_w^*, we find that:

$$\varphi_1 i_l l_w^2 - \varphi_0 w l_w + z^* w = 0 \tag{54}$$

From equation 12 we also have that:

$$h = \frac{v g_z}{u_n} \tag{55}$$

Replacing equation 3 into equation 55:

$$h^* = \frac{v}{u_n}\varphi_0 - \frac{v}{u_n}\varphi_1\frac{i_l}{w}l_w \tag{56}$$

Defining $\varphi_0' = \frac{v}{u_n}\varphi_0$ and $\varphi_1' = -\frac{v}{u_n}\frac{i_l}{w}\varphi_1$, we then have that:

$$h^* = \varphi_0' + \varphi_1' l_w \tag{57}$$

From equation 18 we must have that:

$$z^* = s - h^* - a l_w \tag{58}$$

Replacing equation 57 in the equation above we get that:

$$z^* = s - \varphi_0' - \varphi_1' l_w - a l_w \tag{59}$$

Defining $\beta_0 = s - \varphi_0'$ and $\beta_1 = \varphi_1' + a$, we get that:

$$z^* = \beta_0 - \beta_1 l_w \tag{60}$$

We can also define $\alpha_0' = \frac{\varphi_0 w}{\varphi_1 i_l}$ and $\alpha_1' = \frac{w}{\varphi_1 i_l}$ and rewrite equation 54 as:

$$l_w^2 - \alpha_0' l_w + \alpha_1' z^* = 0 \tag{61}$$

Consequently we have the following system of two equations and two variables:

$$\begin{aligned} z^* &= \beta_0 - \beta_1 l_w \\ l_w^2 - \alpha_0' l_w + \alpha_1' z^* &= 0 \end{aligned} \tag{62}$$

To solve the system above we replace one equation into the other and we get:

$$l_w^2 - \alpha_0' l_w + \alpha_1' \beta_0 - \alpha_1' \beta_1 l_w = 0 \tag{63}$$

Defining $\beta_1' = \alpha_0' + \alpha_1'\beta_1$ and rearranging, we get that:

$$l_w^2 - \beta_1' l_w + \alpha_1'\beta_0 = 0 \tag{64}$$

Where $\beta_1' = \frac{w}{\varphi_1 i_l}(\varphi_0 + a) - \frac{v}{u_n}$ and $\alpha_1'\beta_0 = \frac{w}{\varphi_1 i_l}(s - \frac{v}{u_n}\varphi_0)$, which can also be rewritten as:

$$l_w^2 + bl_w + c = 0 \tag{65}$$

Where $b = -\beta_1' = \frac{v}{u_n} + \frac{w}{\varphi_1 i_l}(i_l(\alpha_1 - \alpha_2) - \varphi_0)$ and $c = \frac{w}{\varphi_1 i_l}(1 - \alpha_1 w - \alpha_2 s_d \pi - \frac{v}{u_n}\varphi_0)$

$$l_w = -b \pm \sqrt{b^2 - 4c} \tag{66}$$

Appendix D - Steady State Dynamics Derivation

Given that:

$$\dot{h} = h\gamma\left[\frac{v[g_l - (\alpha_1 - \alpha_2)i_l]l_{k_w}}{(s-h)} - u_n\right] \tag{67}$$

First, we look for an equation to describe $\dot{l}_{k_w}$:

$$\dot{l}_{k_w} = \frac{\dot{L}_w}{L_w}\frac{L_w}{K} - \frac{L_w}{K}\frac{\dot{K}}{K} \tag{68}$$

$$\dot{l}_{k_w} = l_{k_w}(g_l - g_K) \tag{69}$$

$$\dot{l}_{k_w} = l_{k_w}(g_l - h\frac{u}{v}) \tag{70}$$

Then we look for an equation to describe $\dot{g}_l$:

$$\dot{g}_l = \frac{\dot{Z}L_w}{L_w^2} - \frac{Z\dot{L}_w}{L_w^2} = \frac{Z}{L_w}\left[\frac{\dot{Z}}{Z} - \frac{\dot{L}_w}{L_w}\right] \tag{71}$$

$$\dot{g}_l = g_l(g_z - g_l) = g_l(\varphi_0 - \varphi_1\frac{i_l}{w}l_w - g_l) \tag{72}$$

Finally, to complete the dynamics above we will need one final equation that describes that behavior of l_w. By the definition of l_w, we must have:

$$\dot{l}_w = \frac{\dot{L}_w Y}{Y^2} - \frac{L_w \dot{Y}}{Y^2} = \frac{\dot{L}_w}{Y} - \frac{L_w}{Y}\frac{\dot{Y}}{Y} \tag{73}$$

$$\dot{l}_w = \frac{Z}{Y} - g_Y l_w = z - g_Y l_w \tag{74}$$

$$\dot{l}_w = g_l l_w - g_Y l_w = l_w(g_l - g_Y) \tag{75}$$

Appendix E - Further derivation of the Jacobian matrix and its Routh-Hurwitz determinants

Since, as we have seen our Jacobian matrix is given by:

$$\begin{bmatrix} h_h & h_k & h_g & h_w \\ k_h & k_k & k_g & k_w \\ g_h & g_k & g_g & g_w \\ w_h & w_k & w_g & w_w \end{bmatrix} \tag{25}$$

Where h_h is defined as the partial derivative of $\dot{h}$ with respect to h, h_k with respect to l_{k_w}, h_g with respect to g_l and h_w with respect to l_w. k_x, g_x and w_x represent the partial derivatives of $\dot{l}_{k_w}$, $\dot{g}_l$ and $\dot{l}_w$, respectively. Since it is easy to see that $h_w = 0$, $k_w = 0$, $g_h = 0$ and $g_k = 0$, we can write our $[J - \lambda I]$ matrix as:

$$\begin{bmatrix} (h_h - \lambda) & h_k & h_g & 0 \\ k_h & (k_k - \lambda) & k_g & 0 \\ 0 & 0 & (g_g - \lambda) & g_w \\ w_h & w_k & w_g & (w_w - \lambda) \end{bmatrix} \tag{76}$$

Therefore, $Det[J - \lambda I]$ will be given by:

$$
(g_w) \begin{vmatrix} (h_h-\lambda) & h_k & h_g \\ k_h & (k_k-\lambda) & k_g \\ w_h & w_k & w_g \end{vmatrix} +(\lambda - w_w) \begin{vmatrix} (h_h-\lambda) & h_k & h_g \\ k_h & (k_k-\lambda) & k_g \\ 0 & 0 & (g_g-\lambda) \end{vmatrix} \tag{77}
$$

$$
\begin{aligned}
Det[J-\lambda I] \quad = \quad & g_w\{(h_h-\lambda)[w_g(k_k-\lambda)-k_g w_k]-h_k(k_h w_g - k_g w_h) \\
& +h_g[k_h w_k - w_h(k_k-\lambda)]\} \\
& +(\lambda - w_w)(g_g-\lambda)[(h_h-\lambda)(k_k-\lambda)-h_k k_h]
\end{aligned} \tag{78}
$$

$$
\begin{aligned}
= \quad & g_w\{(h_h-\lambda)[w_g k_k - w_g\lambda - k_g w_k] - h_k k_h w_g + h_k k_g w_h \\
& +h_g k_h w_k - h_g w_h k_k + h_g w_h \lambda\} \\
& [-\lambda^2+\lambda(w_w+g_g)-w_w g_g][h_h k_k - \lambda(h_h+k_k)+\lambda^2-h_k k_h]
\end{aligned} \tag{79}
$$

$$
\begin{aligned}
= \quad & g_w(h_h w_g k_k - h_h w_g\lambda - h_h k_g w_k - \lambda w_g k_k + w_g\lambda^2 + \lambda k_g w_k \\
& -h_k k_h w_g + h_k k_g w_h + h_g k_h w_k - h_g w_h k_k + h_g w_h \lambda) \\
& -\lambda^2 h_h k_k + \lambda^3(h_h+k_k) - \lambda^4 - \lambda^2 h_k k_h \\
& +\lambda(w_w+g_g)[h_h k_k - \lambda(h_h+k_k)+\lambda^2-h_k k_h] \\
& -w_w g_g[h_h k_k - \lambda(h_h+k_k)+\lambda^2-h_k k_h]
\end{aligned} \tag{80}
$$

$$
\begin{aligned}
= \quad & g_w h_h w_g k_k - g_w h_h w_g \lambda - g_w h_h k_g w_k - \lambda g_w w_g k_k + g_w w_g \lambda^2 + \lambda g_w k_g w_k \\
& -g_w h_k k_h w_g + g_w h_k k_g w_h + g_w h_g k_h w_k - g_w h_g w_h k_k + g_w h_g w_h \lambda \\
& -\lambda^2 h_h k_k + \lambda^3 h_h + \lambda^3 k_k - \lambda^4 - \lambda^2 h_k k_h + \lambda(w_w h_h k_k + g_g h_h k_k) \\
& -\lambda^2(h_h+k_k)(w_w+g_g) + \lambda^3(w_w+g_g) - \lambda h_k k_h w_w - \lambda h_k k_h g_g \\
& -w_w g_g h_h k_k + \lambda(h_h w_w g_g + k_k w_w g_g) - \lambda^2 w_w g_g + h_k k_h w_w g_g
\end{aligned} \tag{81}
$$

$$
\begin{aligned}
&\lambda^4 - \lambda^3(w_w + g_g + h_h + k_k) \\
&+\lambda^2(h_h k_k + h_k k_h + h_h w_w + k_k w_w + h_h g_g + k_k g_g \\
&-g_w w_g + w_w g_g) \\
&+\lambda(g_w h_h w_g + g_w w_g k_k + h_k k_h w_w + h_k k_h g_g - h_h w_w g_g - k_k w_w g_g \\
&-g_w k_g w_k - g_w h_g w_h - w_w h_h k_k - g_g h_h k_k) \\
&+g_w h_h k_g w_k + g_w h_k k_h w_g + g_w h_g w_h k_k + w_w g_g h_h k_k - h_k k_h w_w g_g \\
&-g_w h_h w_g k_k - g_w h_k k_g w_h - g_w h_g k_h w_k = 0
\end{aligned} \tag{82}
$$

Which then gives us the following characteristic polynomial:

$$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 \tag{26}$$

Where: $a_0 = 1$; $a_1 = -(w_w + g_g + h_h + k_k)$; $a_2 = (h_h k_k + h_k k_h + h_h w_w + k_k w_w + h_h g_g + k_k g_g + w_w g_g - g_w w_g)$; $a_3 = g_w h_h w_g + g_w w_g k_k + h_k k_h w_w + h_k k_h g_g - h_h w_w g_g - k_k w_w g_g - g_w k_g w_k - g_w h_g w_h - w_w h_h k_k - g_g h_h k_k$ and $a_4 = g_w h_h k_g w_k + g_w h_k k_h w_g + g_w h_g w_h k_k + w_w g_g h_h k_k - h_k k_h w_w g_g - g_w h_h w_g k_k - g_w h_k k_g w_h - g_w h_g k_h w_k$

Appendix F - Derivatives of the Jacobian matrix evaluated at the steady state

$$h_h = \frac{\partial \dot{h}}{\partial h} = \gamma\left[\frac{vs(g_l + a)l_{k_w}}{(s-h)^2} - u_n\right] = \frac{\gamma u_n h^*}{(s-h^*)} \tag{83}$$

$$h_k = \frac{\partial \dot{h}}{\partial l_{k_w}} = \frac{h\gamma v(g_l + a)}{(s-h)} = \frac{h^*\gamma v}{l_w^*} \tag{84}$$

$$h_g = \frac{\partial \dot{h}}{\partial g_l} = \frac{h\gamma v l_{k_w}}{(s-h)} = \frac{h^*\gamma l_w^* u_n}{(s-h^*)} \tag{85}$$

$$k_h = \frac{\partial \dot{l}_{k_w}}{\partial h} = -\frac{s(g_l + a)l_{k_w}^2}{(s-h)^2} = -\frac{u_n^2 l_w^* s}{v^2(s-h^*)} \tag{86}$$

$$k_k = \frac{\partial \dot{l}_{k_w}}{\partial l_{k_w}} = g_l - \frac{2h(g_l + a)l_{k_w}}{(s-h)} = -g_z^* \tag{87}$$

$$k_g = \frac{\partial \dot{l}_{k_w}}{\partial g_l} = l_{k_w}\left[1 - \frac{l_{k_w}h}{(s-h)}\right] = \frac{l_w^* u_n}{v}\left[1 - \frac{l_w^* u_n h^*}{v(s-h^*)}\right] \tag{88}$$

$$g_g = \frac{\partial \dot{g}_l}{\partial g_l} = \varphi_0 - \varphi_1 \frac{i_l}{w} l_w - 2g_l = -g_z^* \tag{89}$$

$$g_w = \frac{\partial \dot{g}_l}{\partial l_w} = -g_l \varphi_1 \frac{i_l}{w} = -g_z^* \varphi_1 \frac{i_l}{w} \tag{90}$$

$$\begin{aligned} w_h = \frac{\partial \dot{l}_w}{\partial h} \quad = \quad & l_w \Bigg\{ -\frac{l_w}{(s-h)^2}\Bigg[g_l(\varphi_0 - \varphi_1 \tfrac{i_l}{w} l_w + a) \\ & + \frac{h\gamma}{(s-h)}(g_l + a)\left(\frac{v(g_l+a)l_{k_w}}{(s-h)} - u_n\right)\Bigg] \\ & - \frac{l_w}{(s-h)}\Bigg[\frac{(s-h)+h}{(s-h)^2}\gamma(g_l+a)\left(\frac{v(g_l+a)l_{k_w}}{(s-h)} - u_n\right) \\ & + \frac{h\gamma}{(s-h)}(g_l+a)\frac{v(g_l+a)l_{k_w}}{(s-h)^2}\Bigg]\Bigg\} \end{aligned} \tag{91}$$

$$\begin{aligned} w_h^* &= -\frac{l_w}{(s-h)}\left\{\frac{l_w(g_l+a)}{(s-h)}g_l + l_w\left[\frac{h\gamma}{(s-h)}(g_l+a)\frac{u_n}{(s-h)}\right]\right\} \\ w_h^* &= -\frac{l_w}{(s-h)}\left\{g_l + \frac{h\gamma u_n}{(s-h)}\right\} \end{aligned} \tag{92}$$

Since $l_{k_w} = \frac{l_w u_n}{v}$ and $u_n = \frac{v(g_l+a)l_{k_w}}{(s-h)}$, which implies that $\frac{l_w(g_l+a)}{(s-h)} = \frac{l_{k_w} v(g_l+a)}{u_n(s-h)} = \frac{l_{k_w}(g_l+a)}{(s-h)}\frac{v}{u_n} = \frac{u_n}{v}\frac{v}{u_n} = 1$

$$w_k = \frac{\partial \dot{l}_w}{\partial l_{k_w}} = -\frac{l_w^2(g_l+a)^2 h\gamma v}{(s-h)^3} = -\frac{h^*\gamma v}{(s-h^*)} \tag{93}$$

$$
\begin{aligned}
w_g = \frac{\partial \dot{l}_w}{\partial g_l} \quad = \quad & l_w \Bigg\{ 1 - \tfrac{l_w}{(s-h)} \Bigg[(\varphi_0 - \varphi_1 \tfrac{i_l}{w} l_w + a) \\
& + \tfrac{h\gamma}{(s-h)} \left(\tfrac{v(g_l+a) l_{k_w}}{(s-h)} - u_n \right) \\
& + \tfrac{h\gamma}{(s-h)} (g_l + a) \tfrac{v l_{k_w}}{(s-h)} \Bigg] \Bigg\}
\end{aligned} \tag{94}
$$

$$
\begin{aligned}
w_g \quad &= \quad \frac{\partial \dot{l}_w}{\partial g_l} = l_w \left\{ 1 - \tfrac{l_w}{(s-h)} \left[(g_l + a) + \tfrac{h\gamma}{(s-h)} u_n \right] \right\} \\
w_g \quad &= \quad l_w \left\{ 1 - 1 - \tfrac{l_w}{(s-h)} \left[\tfrac{h\gamma}{(s-h)} u_n \right] \right\} = - \tfrac{l_w^2 h\gamma u_n}{(s-h)^2}
\end{aligned} \tag{95}
$$

$$
\begin{aligned}
w_w = \frac{\partial \dot{l}_w}{\partial l_w} \quad = \quad & g_l - \tfrac{2 l_w}{(s-h)} \Big[g_l (\varphi_0 - \varphi_1 \tfrac{i_l}{w} l_w + a) \\
& + \tfrac{h\gamma}{(s-h)} (g_l + a) \big(\tfrac{v(g_l+a) l_{k_w}}{(s-h)} - u_n \big) \Big] \\
& + \tfrac{l_w^2}{(s-h)} g_l \varphi_1 \tfrac{i_l}{w}
\end{aligned} \tag{96}
$$

Finally:

$$
w_w^* = g_z \left[1 - \frac{l_w^*}{(s-h)} \left(2(\varphi_0 + a) - \varphi_1 \frac{i_l}{w} l_w^* \right) \right] \tag{97}
$$

$$
w_w^* = g_z \left[1 - \frac{l_w^*}{(s-h)} \left((\varphi_0 - \varphi_1 \frac{i_l}{w} l_w^* + a) + (\varphi_0 + a) \right) \right] \tag{98}
$$

$$
w_w^* = g_z \left[1 - \frac{l_w^*}{(s-h)} \left((g_l^* + a) + (\varphi_0 + a) \right) \right] \tag{99}
$$

And since:

$$
l_{k_w}^* = \frac{L_w}{K} = \frac{l_w^* u_n}{v} \tag{100}
$$

Then:

$$
w_w^* = g_z \left[1 - \frac{l_{k_w}^* (g_l^* + a) v}{(s-h) u_n} - \frac{l_w^*}{(s-h)} (\varphi_0 + a) \right] \tag{101}
$$

$$
w_w^* = -g_z \left[\frac{l_w^*}{(s-h)} (\varphi_0 + a) \right] \tag{102}
$$

Appendix G - Trace of the Jacobian matrix

$$T(J^*) = h^*_h + k^*_k + g^*_g + w^*_w \tag{28}$$

We also know from the previous derivation of the jacobian in steady state that: i)$h^*_h = \frac{\gamma u_n h^*}{(s-h^*)}$; ii) $k^*_k = -g^*_z$; iii) $g^*_g = -g^*_z$; iv) $w^*_w = -g^*_z\left[\frac{l^*_w}{(s-h)}(\varphi_0 + a)\right]$. As a result, we must also have that:

$$\begin{aligned} T(J^*) \quad = \quad & \tfrac{\gamma u_n h^*}{(s-h^*)} - g^*_z\left[\tfrac{l^*_w}{(s-h^*)}(\varphi_0 + a)\right] \\ & -2g^*_z \end{aligned} \tag{30}$$

Since:

$$h^* = \frac{v g^*_z}{u_n} \tag{23}$$

Then:

$$T(J^*) = g^*_z\left[\frac{(\gamma v - l^*_w(\varphi_0 + a))}{(s - h^*)} - 2\right] \tag{103}$$

Then if we assume $g^*_z > 0$, then we must have:

$$\left[\frac{(\gamma v - l^*_w(\varphi_0 + a))}{(s - h^*)} - 2\right] < 0 \tag{104}$$

For $T(J^*) < 0$. If we then replace h^* by its steady state value and we get that:

$$(\gamma v - l^*_w(\varphi_0 + a)) - 2\left(s - \frac{v(\varphi_0 - \varphi_1 \frac{i_l}{w} l^*_w)}{u_n}\right) < 0 \tag{105}$$

As long as $(s - h) > 0$, which we have already assumed as the keynesian stability condition of our model. And since the inequality above can be rewritten as:

$$-l^*_w\left(\varphi_0 + a + 2v\varphi_1 \frac{i_l}{w u_n}\right) + \left(\gamma v - 2(s - v\frac{\varphi_0}{u_n})\right) < 0 \tag{106}$$

So, as long as $g_z^* > 0$ and $\varphi_0 + a + \frac{2v\varphi_1 i_l}{u_n w} > 0$, we will need:

$$l_w^* > \frac{\gamma v - 2\left(s - \frac{v\varphi_0}{u_n}\right)}{\varphi_0 + a + \frac{2v\varphi_1 i_l}{u_n w}} \tag{107}$$

For $T(J^*) < 0$ and, therefore, to have local stability. Finally, considering also that for $g_z^* > 0$, we must have:

$$l_w^* < \frac{\varphi_0 w}{\varphi_1 i_l} \tag{108}$$

Then we conclude that for $T(J^*) < 0$, we must have:

$$\frac{\varphi_0 w}{\varphi_1 i_l} > l_w^* > \frac{\gamma v - 2\left(s - \frac{v\varphi_0}{u_n}\right)}{\varphi_0 + a + \frac{2v\varphi_1 i_l}{u_n w}} \tag{31}$$