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Abstract

This paper describes a hybrid post-Keynesian and classical/neo-Marxian model with a "center of gravity" where the actual, warranted, and natural growth rates coincide. In the model, investment determines saving in the short run, while investment depends on anticipated demand. The Keynesian stability is assumed not to hold, so the model features short-run Harrodian instability, which is bounded by a ceiling and floor. The resulting Kaleckian-Harrodian model is shown to produce some key stylized facts as long-run tendencies, to exhibit wage-led behavior, and to produce depressions in some circumstances.

Keywords: Kaleckian; Harrodian; classical; neo-Marxian; cycles; technological change
JEL codes: B50, E32, O40

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1 Introduction

Post-Keynesian theory has a rich tradition (Hein, 2014; Kurz and Salvadori, 2010). Besides Keynes, important contributors include Kalecki, Kaldor, Harrod, and Robinson (Lavoie, 2015, Fig. 1.1). While all post-Keynesian theory retains Keynes' argument that output is determined by the level of effective demand, that rich diversity offers sometimes conflicting explanations of economic phenomena.

Theoretical divergence has been a source of both inspiration and consternation to contemporary post-Keyensian theorists. In the search for a common core, the neo-Kaleckian family of models, first proposed by Del Monte (1975), has performed well. It explains underutilization of capital and cleanly illustrates the paradoxes of thrift and of costs (Lavoie, 2014). Work by Rowthorn (1981), Dutt (1984, 1987), Taylor (1985), Blecker (1989), and Bhaduri and Marglin (1990), among others, considerably expanded Kalecki's original ideas to cover stagnation, "exhilaration", and open economies. Many of these results follow from an assumption that "Keynesian stability" holds; that is, that savings responds more strongly than does investment to a change in capacity utilization.

With greater prominence, the neo-Kaleckian model drew closer scrutiny, and – perhaps unavoidably – criticism (Lavoie, 1995; Hein et al., 2011). A major point of contention is whether capacity utilization can deviate persistently from the level desired by firms. In standard presentations of the neo-Kaleckian model, utilization can remain at a low level indefinitely, because it is determined in part by saving behavior, and thus is out of the direct control of firms. Yet, as Skott (2012) points out, firms in fact have a substantial measure of control over the factors that determine long-run utilization, so there is no practical reason they should be unable to meet (or at least continually move towards) their target. And firms are aware of their potential utilization; Corrado and Mattey (1997) argue that "those who discuss production capability with plant managers quickly discover that managers generally are quite precise about how much their facilities can produce without extraordinary efforts."

Kurz (1986) argues that firms ought to target the least-cost level of utilization as part of their choice of technique, a proposition that appears consistent with observed micro-level firm behavior (Mattey and Strongin, 1997). Building on this idea, Nikiforos (2013) created a model in which utilization is determined by a cost-minimizing firm that was subsequently applied to a Kaleckian analysis by Nikiforos (2016) and Dávila-Fernández et al. (2019).

If target utilization is exogenous, then a natural parameter to adjust endogenously in order to meet it is the "animal spirits" term in an investment function or, in the terminology of Hicks (1950), "autonomous investment", which reflects firms' medium-run to long-run demand expectations. Yet, Kaleckian models with adaptive expectations models for autonomous investment are unstable as long as the Keynesian stability condition holds. The sources of the instability and some proposed solutions are reviewed and critiqued by Hein et al. (2011) and Girardi and Pariboni (2019); from these papers, and the response by Lavoie (2019) to Dávila-Fernández et al. (2019), it is reasonable to conclude that neither Kaleckians nor Harrodians feel convinced by the mechanisms proposed by their counterparts in the debate.

One possibility that has been explored by a small number of authors is to abandon the Keynesian stability condition in order to tame long-run Harrodian instability. Skott (2012) proposes such a "Kaleckian-Harrodian" model, in which the Keynesian stability condition holds in the short run but not the long run. Fazzari et al. (2013) propose a model in which the Keynesian stability condition does not hold even in the short run. We follow the latter approach, accepting Harrod's argument that investment responds more strongly than saving to shifts in utilization in the short run, thus leading to instability (Harrod, 1939). As this stabilizes adaptive expectations of autonomous investment, the result, somewhat paradoxically, is long-run Harrod *stability*, as the actual growth rate tends towards the warranted rate (Fazzari et al., 2013, 2018). Thus, abandoning the Keynesian stability condition resolves Harrod's unstable dynamics in the long run. The price of that result is unstable Hicksian accelerator-multiplier dynamics (Hicks, 1950) in the short run.

On its face, the evidence appears to be against this position. By the early 1970s, the consensus view of decades of business cycle studies was that endogenous saving and expenditure dynamics are stable, so cycles must be the result of external shocks from which the economy subsequently recovers (Hymans, 1972). However, Blatt (1978) showed that the econometric tests in use at the time, which assumed a linear model, would falsely suggest stable dynamics even when the underlying process was unstable and nonlinear. The well-known asymmetry of business cycles, with short contractions and long expansions, is *prima facie* evidence of nonlinearity. Subsequent econometric tests have provided additional evidence of nonlinearity for business cycles in the US and Europe (Brock, 1991; Teräsvirta and Anderson, 1992; Pesaran and Potter, 1997; Clements and Krolzig, 1998; Razzak, 2001; Belaire-Franch and Contreras, 2003). These findings support those of Skott (2012), who found, in contrast to Lavoie et al. (2004), that empirical estimates of parameters for a Kaleckian model specification imply unstable rather than stable dynamics.

In this paper we combine the mechanism explored by Fazzari et al. (2013) to bring the actual and warranted rates into alignment with a further classical (or neo-Marxian) mechanism to endogenously bring the natural and actual rates into alignment (Foley, 2003; Julius, 2005; Shaikh, 2016, p.652). The relevant processes are cost share-induced technological change combined with employment-dependent labor costs. In this process, productivity growth depends on the functional income distribution, which is in turn influenced by productivity. The result is a stabilizing mechanism that aligns the actual and natural rates. A recurring finding in this literature is that this mechanism damps Goodwin (1951) cycles (Foley, 2003; Julius, 2005). In this paper, a Goodwin-type mechanism drives labor costs, while cycles arise from the Harrodian instability.

The main novelty of this paper is to demonstrate how the combination of these two mechanisms leads to convergence of the actual, warranted, and natural growth rates as a long-run tendency while generating cycles of different lengths. We show these results using a simulation model that is based on US data. The model is under active development. A prior, 5-sector, version is documented in Kemp-Benedict (2017b) and an updated version described in Kemp-Benedict and Ghosh (2018). The current version has 12 sectors and has been further modified. We focus on those aspects of the model that relate to debates between Kaleckian and Harrodian perspectives. The model is post-Keynesian in that investment demand determines saving in the short run, while investment decisions are made on the basis of anticipated demand. It draws on Kaleckian, Harrodian, Sraffian, evolutionary, and Marxian theory, and yields Kaldorian outcomes as long-run tendencies.

The model presented in this paper has a number of interconnected "moving parts". The first half of the paper takes some of those parts in turn and explains how they contribute both to cycles and long-run stability. The second half presents results from the model and shows that it can reproduce some key stylized facts as identified by Taylor et al. (2018). The model exhibits a kind of "wage-led growth" and can, despite the stabilizing mechanisms, enter into a deep slump that the endogenous dynamics may not be able to overcome. We also reproduce Blatt's theoretical result, showing that the locally unstable model yields spuriously stable coefficients when fit to a linear regression model.

2 A Kaleckian-Harrodian framework

The full simulation model, which contains multiple sectors and processes, is presented in summary form later in the paper. In this section we present some of the main features using simplified analytical models.

We begin with a neo-Kaleckian model with an investment function

$$g^i = \gamma + \alpha\,(u - u^*)\,, \tag{1}$$

and a saving function

$$g^s = s_0 u^* \kappa + s_1\,(u - u^*)\,\kappa. \tag{2}$$

In these equations, u is capacity utilization, while u^* is a target utilization level. In the language of Hicks (1950), γ is "autonomous" investment, while $\alpha\,(u - u^*)$ is "induced" investment. The coefficient s_1 is the marginal change in saving with a change in utilization and κ is capital productivity. We refer to s_0 as "autonomous" saving, but emphasize that, like Hicks' autonomous investment, it is not purely autonomous. Rather, it exhibits inertia because it depends on comparatively slowly-changing factors. It is related to Serrano's autonomous consumption term through the relationship

$$\text{autonomous consumption per unit of capital stock} = (1 - s_0)u^*\kappa. \tag{3}$$

According to Harrod (1939), the warranted rate of growth g_w is "that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount." That is, it is the rate of growth corresponding to planned (or *ex ante*) saving at the target level of utilization. Thus, $g_w = s_0 u^* \kappa$. The actual rate of growth is the one that obtains at the level of utilization at which investment equals saving *ex post*. Setting Eqn. (1) equal to Eqn. (2), we find that level of utilization to equal

$$u = u^* + \frac{\gamma - s_0 u^* \kappa}{s_1 \kappa - \alpha}. \tag{4}$$

The denominator in the second term, $s_1\kappa - \alpha$, which we write as σ, is the gap between the marginal change in saving and the marginal change in investment to a change in utilization. When it is positive, the Keynesian stability condition holds; when it is negative, there is short-run Harrodian instability.

Next, we introduce adaptive expectations for γ. First, firms are assumed to assess how close utilization has been to the target (the utilization gap) over a recent medium run, smoothing over short-run fluctuations. We represent this behavior using a variable z that obeys

$$z = (1 - \theta)\,z_{-1} + \theta\,(u - u^*)\,. \tag{5}$$

Firms then anchor their expectations to the level of investment they would make if the realized utilization gap were equal to its recent average as captured by z. That is,

$$\gamma = \gamma_{-1} + \alpha z_{-1}. \tag{6}$$

Substituting for u in Eqn. (5) using Eqn. (4) gives an expression for the change in z,

$$\Delta z = -\theta z_{-1} + \frac{\theta}{\sigma}\,(\gamma - g_w)\,, \tag{7}$$

where we have substituted for the warranted growth rate $g_w = s_0 u^* \kappa$. From Eqn. (6), the change in γ is given by

$$\Delta\gamma = \alpha z_{-1}. \tag{8}$$

At equilibrium, $\gamma = g_w$ and $z = 0$. That is, at equilibrium the actual growth rate is equal to the warranted rate. We now ask whether the equilibrium is stable.

2.1 Convergence of the actual to the warranted growth rate

Throughout most of the paper we work in discrete time – the simulation model operates on a quarterly time step. Discretization can introduce instabilities not present in continuous-time models, depending on the details of assumed behavior. In this section, we abstract from those details and study stability in continuous time, noting that additional instabilities can arise in the real world (and in the simulation model) due to delayed feedback and discontinuous, "lumpy" actions.

The continuous-time version of Eqns. (7) and (8) can be written as a matrix equation. Using a dot to represent a time derivative, we have

$$\begin{bmatrix} \dot{z} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} -\theta & \theta/\sigma \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} z \\ \gamma \end{bmatrix} + \begin{bmatrix} -(\theta/\sigma)g_w \\ 0 \end{bmatrix}. \tag{9}$$

The equilibrium of this system is stable as long as the trace of the square matrix of coefficients is negative and the determinant is positive. They are given by

$$\mathrm{Tr} = -\theta, \tag{10a}$$

$$\mathrm{Det} = -\frac{\alpha\theta}{\sigma}. \tag{10b}$$

The trace is unambiguously negative, but the determinant is positive only if σ is negative. That is, long-run stability is achieved at the expense of short-run instability.

Can we make this assumption? Debates over business cycle models through the 1970s centered on two distinct types of models: a) stable models that generate cycles only when disturbed by exogenous shocks (as in Frisch, 1933) and b) unstable multiplier-accelerator models bounded by a ceiling and floor (as in Hicks, 1950). A third type was generally not discussed: c) stable models that generate endogenous cycles (in particular Goodwin, 1951). Econometric tests and model simulation experiments appeared to suggest that type (a) was most likely. Hymans (1972, pp. 539-540) argued that "the weight of reasoned evidence" supported the idea that, "The business cycle is not endogenous; rather it is the result of a normally stable, or damped, system reacting to external influences." Adding weight to the econometric evidence, he thought it highly unlikely that the business sector, characterized as it is by highly concentrated markets, should repeatedly fail to anticipate the consequences of its own internal dynamics. However, while markets are concentrated, the economy is linked through a web of intermediate exchange that is less easy to anticipate.

Blatt (1978, 1980) pointed out that the econometric tests that Hyman was responding to would give a false positive for models of type (a) if the underlying process was nonlinear. Nonlinear dynamics are quite plausible, because only nonlinear processes can generate asymmetric cycles, and business cycles are known to be asymmetric, with short contractions and long expansions. The same critique applies to the Goodwin model, which is a linear model with symmetric cycles. Econometric tests that are sensitive to asymmetry consistently suggest asymmetric cycles for Europe and present mixed findings for the US

(Teräsvirta and Anderson, 1992; Pesaran and Potter, 1997; Clements and Krolzig, 1998; Razzak, 2001; Belaire-Franch and Contreras, 2003). Blatt's arguments and the subsequent tests (including those of Skott, 2012) at least open the possibility for Hicksian multiplier-accelerator dynamics.

2.2 Convergence of the natural to the actual growth rate

We have shown, in agreement with Fazzari et al. (2013, 2018), that short-run Keynesian instability is consistent with long-run Harrodian stability, as adaptive expectations for autonomous investment cause the actual growth rate to converge on the warranted rate. A further dynamic in the model presented in this paper brings the natural growth rate in line with the actual growth rate. It rests on three processes: conflict-based wage determination (e.g., as in Goodwin, 1967; Rowthorn, 1977); cost share-induced technological change; and target-return pricing. The first two are common features of neo-Marxian models, while Lavoie (1996) argues that target-return pricing is a reasonable assumption for modern firms.

In this model we apply a classical-evolutionary theory of cost share-induced technological change developed in Kemp-Benedict (2019) that generalizes a model proposed by Duménil and Lévy (1995, 2010). A simplified derivation of the core result is presented in the Appendix, along with an extended discussion of theories of cost share-induced technological change. An important implication of the theory is that both labor and capital productivity must respond simultaneously when cost shares change.

After firms respond to costs through marginal changes in capital and labor productivity, their innovations diffuse throughout the economy, after which they adjust their wages and prices to remain competitive. Most neo-Kaleckian models assumed a fixed markup, but Lavoie (1996) argues convincingly that target-return pricing is a more plausible (or at least equally plausible) assumption for modern firms. We therefore assume target-return pricing. As argued by Kemp-Benedict (2019) and shown below, this assumption yields Kaldor's stylized fact of constant capital productivity as a long-run tendency.

We start with an identity. With a capital productivity κ, labor productivity λ, and labor supply L_s, the labor participation rate $e = L/L_s$ is given in terms of the capital stock K as

$$e = \frac{\kappa}{\lambda}\frac{uK}{L_s}. \tag{11}$$

Using a "hat" to represent a growth rate, denoting the growth in the labor supply by $n = \hat{L}_s$, and the actual growth rate by $g_a = \widehat{uK}$, growth in the participation rate is given by

$$\hat{e} = \hat{\kappa} - \hat{\lambda} + g_a - n. \tag{12}$$

Under cost share-induced technological change, a fall (rise) in the profit share π will slow (accelerate) capital productivity growth and accelerate (slow) labor productivity growth. Combined with Eqn. (12) for the growth in the participation rate, we have the following causal sequence:

$$e\uparrow \underset{(a)}{\Rightarrow} \pi\downarrow \underset{(b)}{\Rightarrow} \hat{\kappa}\downarrow, \hat{\lambda}\uparrow \underset{(c)}{\Rightarrow} \hat{e}\downarrow. \tag{13}$$

Step (a) is conflict-based wage determination; step (b) is cost share-induced technological change; step (c) follows from the identity in Eqn. (12). The end result is that a rise in the participation rate e triggers a negative feedback. It is therefore a stabilizing dynamic. The feedback comes after a delay, so it will tend to generate oscillations.

Target-return pricing generates an additional feedback, as noted in Kemp-Benedict (2019). Under target-return pricing, the profit rate $r = \pi\kappa$ is constant in the short run. Given a capital productivity, firms adjust their markup, and therefore the profit share, to meet a target rate of profit. In that case,

$$\hat{r} = 0 \quad \Rightarrow \quad \dot{\pi} = -\pi\hat{\kappa}. \tag{14}$$

This generates a further causal sequence,

$$\hat{\kappa}\uparrow \quad \Rightarrow \quad \pi\downarrow \quad \Rightarrow \quad \hat{\kappa}\downarrow. \tag{15}$$

This is also a stabilizing dynamic, as it features a negative feedback.

Due to the combination of these two stabilizing dynamics, the (stable) equilibrium is characterized by $\hat{e} = \hat{\kappa} = 0$. At that point, from Eqn. (12), we have

$$g_a = \hat{\lambda} + n \, (= g_n). \tag{16}$$

Thus, as labor productivity growth adjusts, it brings the natural growth rate g_n in line with the actual growth rate g_a.

2.3 Additional stabilizing mechanisms

In the model presented in this paper, and like that of Fazzari et al. (2013, 2018), Harrodian instability is contained from above by supply constraints.[1] In principle, any constraint might apply, whether on capital utilization, employment, critical raw materials (such as energy or, in an agricultural economy, the harvest), foreign exchange to buy intermediate and investment goods, or any other necessary input to production.

In practice, post-Keynesian models generally assume capital and labor as inputs. Most Kaleckian models implicitly or explicitly assume abundant labor, so output at full capital utilization is the upper limit to production. In contrast, Skott (2010) proposed a Kaldorian model in which labor supply constrains output. In justifying the Kaldorian closure, he argues that Keynesian theorists overstate the case for price rigidity. We follow a middle path in which markup pricing is the norm, but firms may raise their prices at high levels of utilization.

2.3.1 Supply constraints and price adjustment

We follow neo-Kaleckian theory by assuming that firm investment decisions depend on capacity utilization, profitability, and borrowing costs (Fazzari and Mott, 1986). We further assume that firms adopt a target markup that they may not achieve in any given time period. Firms adjust their prices slowly, so the realized markup lags the target. As firms raise their prices, they are passed on as intermediate costs in downstream sectors, which gradually adjust their own prices in response (a Sraffian mechanism; see Steedman, 1992; Kemp-Benedict, 2017a). As noted above, we further assume that, while firms do not know the demand schedule for their products, they nevertheless adjust their prices in response to changes in capacity utilization. Specifically, sectoral markups depend on utilization through a "hockey-stick" curve that is flat or slowly varying for most utilization levels while rising

[1] An alternative that is arguably better aligned with Harrod's thinking is for firms to form an expectation for a "normal" range for capacity utilization and to act when they are near the edges of that range (Setterfield, 2019).

sharply near full utilization. We expect prices to be more responsive for more competitive sectors and for commodities. The natural resource sectors (mining and agriculture), utilities, and construction are assumed to be particularly responsive.[2]

Wage growth in the model depends on the employment level through a Goodwin-type mechanism (Goodwin, 1951). At a nominal "normal" employment rate, wage growth equals labor productivity growth plus a cost-of-living adjustment based on the smoothed consumer price inflation rate. Wage growth is faster (slower) than labor productivity growth when employment is above (below) the normal level. As with the cost of intermediates, changes in wages propagate through the system of inter-industry exchange as firms gradually adjust their prices. In a straightforward extension of conventional post-Keyensian models, we allow for labor supply to respond to a change in employment levels through changes in the participation rate. The size of the working-age population provides the ultimate constraint on labor supply, while the labor force (slowly) expands and shrinks.

Both for capital utilization and employment, demand can fluctuate rapidly, while supply capacity responds slowly. Over both time scales, the main response involves changes in quantity. In the short run, reduced demand leads to reduced capacity utilization and layoffs. In the medium run, firms observe their utilization levels and cut back on planned investment, while some workers decide to exit the labor force. Accompanying those quantity adjustments are price changes that propagate through the system of inter-industry exchanges. In the short run, the Goodwin model drives changes in the real wage. In the longer run, rising labor costs drive labor-saving innovation (see the Appendix for details).

2.3.2 Stabilization through quasi-autonomous expenditure

Following Serrano (1995a,b), some authors have sought to stabilize neo-Kaleckian models by introducing non-capacity creating autonomous expenditure (Lavoie, 2016; Palley, 2018). Autonomous non-capacity creating expenditure generates induced consumption through a multiplier and induced investment through an accelerator, the combination forming the Sraffian super-multiplier.

In developing the model described in this paper, it was found that some degree of reliability in the growth of consumption was necessary to keep the economy on track. The bulk of expenditure in the model comes from households spending their wages, while firms plan their investment in order to meet expected growth. As Minsky (2008, p. 26) argued, this view of consumption is more consistent with Keynes than is a life-cycle model. In the language of Hicks (1950), the steadier components of demand, dominated by consumption expenditure, largely determine autonomous investment, while fluctuations in utilization, profitability, and the interest rate largely determine induced investment.

The model features two sources of quasi-autonomous non-capacity creating expenditure that introduces some inertia into the economy. The first, which we refer to as "committed consumption" consists of regular consumption expenditure either for food or to meet obligations from previous consumption choices. These include rent or mortgage payments and related household costs, car insurance and gasoline, and education costs. To these we add medical expenditure, which is largely non-discretionary. Committed consumption makes a substantial contribution to total expenditure. Using data from the 2005 US Consumer Expenditure Survey (and thus prior to the Great Financial Crisis) we found that three-quarters

[2]Volatility in commercial construction costs is a chronic challenge (Weidman et al., 2011). For the other sectors, evidence is provided by Clark (2001), who found that prices of food and utilities were comparatively volatile (standard deviation > 9%), while those of most manufactures and services were less volatile.

of expenditure across all households was devoted to food at home (a proxy for habitual food expenditure), housing and related items, transport and related items, healthcare, education, and pension and social security payments.

Committed consumption expenditure is ultimately constrained by wage growth. As acknowledge by Lavoie (2016, p. 194), no component of consumption can be fully autonomous in the long run. The Sraffian super-multiplier is a simplifying assumption in models that seek to study the response of output and investment to consumption demand. In the model presented in this paper, we separate consumption for most goods and services into a quasi-autonomous part that depends on long-run changes in the wage bill from a flexible part that responds to short-run changes in the wage bill.

We assume education and health care expenditure (which are combined in a single sector) to grow in line with total output. These sectors provide a substantial component of employment, but as they supply final household demand, rather than intermediate demand, they are largely disconnected from the system of inter-industry exchanges. If demands for education and health care services were not anchored to GDP, then small variations in employment in other sectors would affect demand for these sectors' outputs, which would then amplify the change in employment. The result would be wide variation in utilization independent of changes in other sectors. Recasting demand for education and health services as a quasi-autonomous source of non-capacity generating demand serves to stabilize the sectoral dynamics through a modified Sraffian super-multiplier mechanism.[3]

3 The model

In this paper, we build upon a multi-sector post-Keynesian model first presented in Kemp-Benedict (2017b) with improvements documented in Kemp-Benedict and Ghosh (2018). The key components are shown in Fig. 1. The present paper provides a summary of the concepts relevant for our analysis.

The model was calibrated to produce business cycles as illustrated by changes in capacity utilization. In this section, we describe the main model components used to determine capacity utilization, starting with demands (investment and consumption), followed by income (wages and profits) and ending with the process of model closure.

3.0.1 Investment

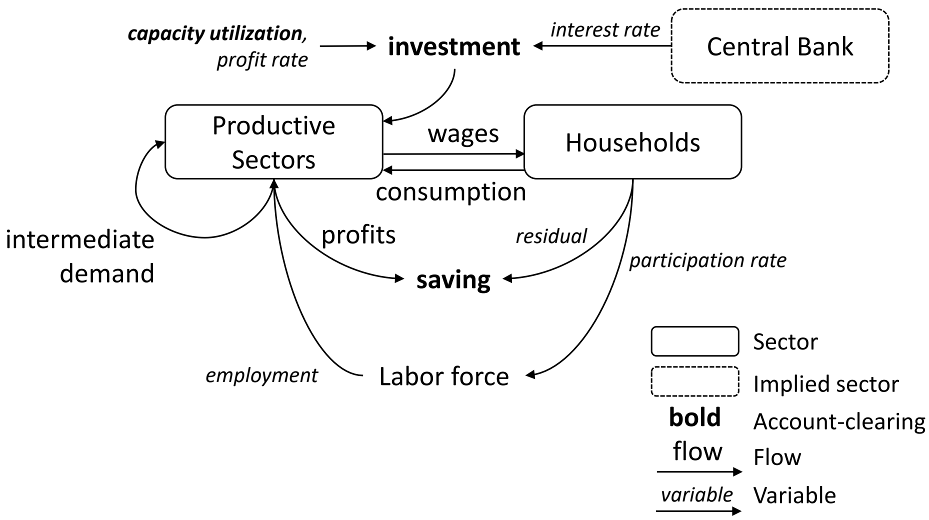
The model is demand-driven, consistent with post-Keynesian principles, with anticipated consumption demand driving investment, which then triggers the production process. Following Hicks (1950), the gross investment rate g_i for sector i has both autonomous and induced elements,

$$g_i = g_i^{\mathrm{ind}} + g_i^{\mathrm{aut}}. \tag{17}$$

High capacity utilization u_i signals rising demand, so it has a dominating influence on induced investment decisions. Consistent with the empirical results of Fazzari and Mott (1986), profits as measured by the profit rate r_i and interest payments as proxied by the

[3] This admittedly oversimplifies the nature of education and health care expenditure. Demand for educational and health care services generally grows in line with population, but education has a counter-cyclical component as laid-off workers return to school. Furthermore, the nature of the services change, as higher incomes allow households to afford more expensive services. We reduce this complex reality to the admittedly oversimplistic assumption that the combined demand for education and health care grows in proportion to real GDP.

Figure 1: Model structure (from Kemp-Benedict and Ghosh, 2018).



interest rate i also influence induced investment. In a slump, the gross investment rate is bounded below by the depreciation rate. The model assumes that not all firms cease investing in a slump, so the lower bound is a fraction f of depreciated capital even during an economic downturn, constituting a depreciation "floor", as proposed by Hicks (1950). The "ceiling" occurs at full capacity utilization; as discussed above, it is also reflected in rising wages and prices. In some sectors, particularly construction, utilization rises above $u = 1$ at the peak. We allow for this because "full utilization" as reported in national statistics represents operation at maximum sustainable production levels, while for brief periods utilization can exceed its sustainable maximum. Induced investment is calculated relative to exogenously-specified normal utilization rate u^*, target rate of return r^* and real neutral rate of interest i^*, as determined by a Taylor (1993) rule,

$$g_i^{\text{ind}} = \max\left[\alpha_i(u_i - u^*) + \beta_i(r_i - r_i^*) + \gamma(i - i^*), -f\delta_i\right]. \tag{18}$$

In the present version of the model we assume the coefficient γ to be the same across sectors, while other parameters are allowed to vary between sectors.

Unlike induced investment, autonomous investment is independent of current output and reflects the long-run prospects of firms. In the model, autonomous investment follows historical trends with a bias, B. Historical trends are represented by the current level of autonomous investment and smoothed induced investment (lagged by one period). Using angle brackets to indicate a smoothed value,

$$g_i^{\text{aut}} = g_{i,-1}^{\text{aut}} + \langle g_i^{\text{ind}} \rangle_{-1} + B. \tag{19}$$

The bias allows for expectations of future output that are not justified by observed historical trends. As Beckert (2016) has noted, this is an important factor explaining economic growth. It is also necessary for the economy to be resilient. Otherwise, as we show in the results, a downturn can turn into a depression.

We allow for capital to become "inactive" during an economic slump due to plant closures or insufficient demand. An increasing fraction of the capital is deactivated as capacity utilization drops below the normal level u^*. Some inactive capital is subsequently reactivated to meet demand for investment goods. Active capital stocks grow when investment I_i in sector i is greater than the rate of depreciation, and both active and inactive capital depreciate at the same rate. After the gross investment rate is determined, orders for investment goods are placed in proportion to the amount of "active" capital,

$$I_i = g_i K_i^{\text{active}}. \tag{20}$$

Production of investment goods occurs after an exogenously specified delay, so new orders tend to cause a backlog. Total investment demand is allocated to producing sectors based on the sectoral share of private investment demands (as determined from the "make" tables in national input-output accounts).

3.0.2 Pricing

Firms set a target price based on a target rate of return (Lavoie, 1996) that varies by sector. The target price changes in response to changing costs of inputs. Sector prices gradually adjust towards the (moving) target in a price updating rule that sets the current-period price to a weighted average of the previous-period price and the target (see Kemp-Benedict, 2017a).

The price mechanism is affected by utilization in two ways. First, following Rowthorn (1977), we assume that firms find it more difficult to change prices when utilization is low. We capture this assumption by giving more weight to the current price than the target when utilization falls below the normal level. Second, as discussed in the background section on pricing, we assume that firms raise their prices at high utilization according to a "hockey-stick" curve that is flat for most utilization levels but that rises more or less sharply as the sector approaches full utilization. Manufacturing is the least responsive, while agriculture, mining, utilities and construction are most responsive.

3.0.3 Consumption and saving

The current version of the model does not track household wealth, so the distribution of profits between retained earnings, interest payments and dividends is not specified. Consumption is based on wages, but may exceed or fall short of the wage bill. The total wage bill W is split into a fixed component for permanent employees and a flexible component that depends on utilization,

$$W = W_{\text{fix}} + \boldsymbol{W'} \cdot \boldsymbol{u}. \tag{21}$$

The wage at normal utilization, W_{norm}, is the value that W takes when $u_i = u^*$.

Personal consumption $\boldsymbol{F}_c$ is divided into base consumption and marginal consumption,

$$\boldsymbol{F}_c = \boldsymbol{F}_{c0} + \boldsymbol{F}_{c1}. \tag{22}$$

As discussed earlier in the paper, base or "committed" consumption reflects the fixed portion of household expenditures, such as food, housing or transportation expenses. Total real base consumption rises with the real wage bill at normal levels of utilization and employment,

W_{norm}, with an adjustment for inflation. To get real consumption, the wage is deflated using the consumer price index P_c, which is calculated as a Laspeyres index,

$$\boldsymbol{F}_{c0} = \frac{W_{\text{norm}}}{P_c}\boldsymbol{f}_{c0}. \tag{23}$$

Marginal consumption represents the flexible portion of household spending beyond committed expenditures. Unlike base consumption, which depends on the normal wage bill, the marginal consumption rate will fluctuate depending on capacity utilization through its impact on employment. Total marginal consumption is given by the marginal consumption rate multiplied by any remaining wage income after normal expenditure. If none remains, consumption falls below the normal level by an extent that depends on utilization relative to the normal level,

$$P_c\boldsymbol{F}_{c1} = \begin{cases} (W - \boldsymbol{p}' \cdot \boldsymbol{F}_{c0})\,\boldsymbol{f}_{c1}, & W > \boldsymbol{p}' \cdot \boldsymbol{F}_{c0}, \\ -\boldsymbol{W}'_{\text{flex}} \cdot (\boldsymbol{u}^* - \boldsymbol{u})\,\boldsymbol{f}_{c1}, & \text{otherwise.} \end{cases} \tag{24}$$

3.0.4 Model closure

Kaleckian models are closed by finding the level of utilization at which saving is equal to investment. We follow a similar strategy for the model presented in this paper, but with two differences. First, we extend the Kaleckian closure to a multi-sector setting. Second, in keeping with the Hicksian features of the model, we constrain output when capital utilization is at or above full capacity.

The multi-sector extension of the Kaleckian closure is conceptually straightforward. As described above, consumption of most sectors' products depends on current-period utilization through the flexible portion of the wage bill. The exception is demand for education and health care, which rises with GDP. Furthermore, demand for intermediate goods and services depends on capacity utilization through its influence on productive activity. Investment demand depends on utilization in previous periods, but as it represents prior orders for investment goods, it is considered fixed in the current period. The result is a system of equations that depends linearly on the vector of sectoral utilization. The coefficients depend on the relationship between utilization, wages and consumption on the demand side, and capital productivity and input-output coefficients on the supply side.

The system can be solved using standard methods from linear algebra, but utilization levels calculated this way can exceed the maximum sustainable level. When they do, household consumption is assumed to be curtailed. This introduces a second-stage correction to the solution of the linear system of equations. The correction reduces utilization levels in all sectors through impacts on wages and intermediate consumption. The reduced utilization then triggers a downturn.

3.1 Implementation

The model was built using Vensim DSS system dynamics modeling software and runs at a quarterly time step. (The code is available from the author upon request.) Model parameters were estimated in Excel using historical US data. We emphasize that despite the use of US data, the model is not representative of the US economy since it does not include the government or financial sectors and is closed to trade.

The model has 12 sectors: agriculture, mining, utilities, construction, manufacturing, wholesale trade, retail, transportation and warehousing, information, professional and business services, educational and healthcare services, and leisure and hospitality services. Intermediate demands were estimated using the 2015 15-sector make and use tables from the Bureau of Economic Analysis (BEA) input-output accounts[4]. A symmetric 12-sector input-output table was developed using the industry-technology assumption (ITA) approach documented in Guo et al. (2002). Selected sectors were excluded from the analysis, including the Finance, Insurance, and Real Estate (FIRE) sector, the Government, and the "Used" and "Other" sectors. Data for the leisure and hospitality sectors are combined.

The size of the labor force was calculated using demographic data from the UN Population Division's World Population Prospects database[5]. From the Bureau of Labor Statistics (BLS)[6] Table 17 we obtained employment data by sector and occupation for all sectors except transportation and warehousing, on the one hand, and the utilities sector, on the other, which have combined employment estimates. We used BLS Table 18 to obtain separate employment data for these two sectors, and found occupational shares from Data USA[7]. An initial labor participation rate of 75% was based on a calculated total employment value and observed output. Base consumption rates are proportional to the total wage bill and observed consumption rates. Marginal consumption rates are a fraction of the base consumption rates, with different multipliers for each sector.

The system of equations for cost share-induced technological change features an equilibrium, as discussed earlier. Labor productivity growth rates at equilibrium were set to 5%/year in the manufacturing sector, 2%/year in the services sectors and 3%/year in all other sectors. Sectoral gross investment and depreciation rates were computed using BEA data for capital stocks (represented by private fixed assets) and flows[8]. We computed initial capital stocks using a specified rate of return of 10% for all sectors, an altered demand vector aligned with the closed-economy assumption, reference employment and utilization rates (set at 95% and 85%, respectively), and the calculated initial labor participation rate. Target profit rates for each sector were estimated from the depreciation rate and assumed rate of return.

These procedures fix some parameters, but leave several others free. The free parameters were adjusted to give persistent cycles.

4 Results

We ran the model for 75 years assuming constant populations. Capacity utilization for the construction, manufacturing, and education and health care sectors are shown in Fig. 2. The trajectories for construction and manufacturing are distinctly asymmetric, reflecting the underlying nonlinear dynamics. In some cycles, capacity utilization in the construction sector exceeds the sustainable level at the peak. As discussed earlier, we allow for unsustainably high utilization over brief periods. In the model, high manufacturing utilization

[4] `https://www.bea.gov/industry/io_annual.htm`, accessed April 6, 2018.

[5] World Population Prospects: The 2017 Revision, custom data acquired from `https://esa.un.org/unpd/WPP/DataQuery/`.

[6] 2015 Annual Averages - Household Data - Tables from Employment and Earnings. `https://www.bls.gov/cps/tables.htm`. Accessed July 25, 2018.

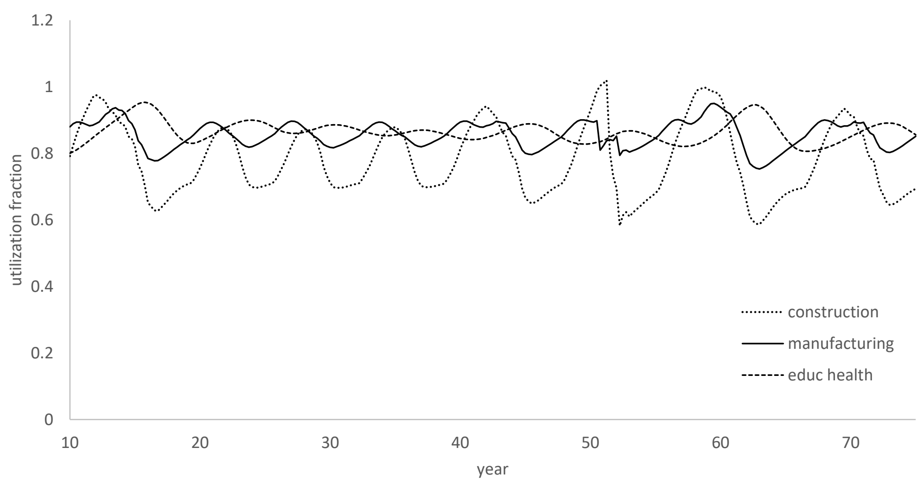
[7] Transportation & Warehousing and Utilities: `https://datausa.io/profile/naics/48-49,%2022/`. Accessed July 25, 2018.

[8] National Data: List of Fixed Assets Accounts tables. `https://www.bea.gov/`. Accessed July 20, 2018.

drives investment in the manufacturing sector, so there is a backlog of investment projects that continues to drive construction activity upward even as manufacturing enters a slump. Cycles in education and health care lag both manufacturing and construction; they are also much smoother. As described above, they are a quasi-autonomous source of demand and execute smooth cycles like those identified by Lavoie (2016). Thus, in this Kaleckian-Harrodian model, cycles bounded by a Hicksian ceiling and floor coexist with regular cycles driven by quasi-autonomous demand.

Following Blatt (1978, 1980), we expect that, due to nonlinearities, estimates of parameters for the investment function from standard econometric tests will be substantially different than the ones actually used in the model. That is the case. We carried out a variable autoregression (VAR) on the investment rate in the manufacturing sector and found a coefficient on utilization of 0.08, on the rate of return of 0.19, and on the interest rate of 0.11. In constrast, the model values were, respectively, 2.00, 0.50, and 0.25. This illustrates Blatt's argument that the results of econometric tests of the business cycle, as summarized by Hymans (1972), are likely to be in error if the underlying dynamics are nonlinear.

Figure 2: Utilization for the construction, manufacturing, and education and health care sectors over 65 years at constant population



Next, we discuss the results with reference to a set of "stylized facts" provide by Taylor et al. (2018). Their list, which expands on that of Kaldor (1957), includes

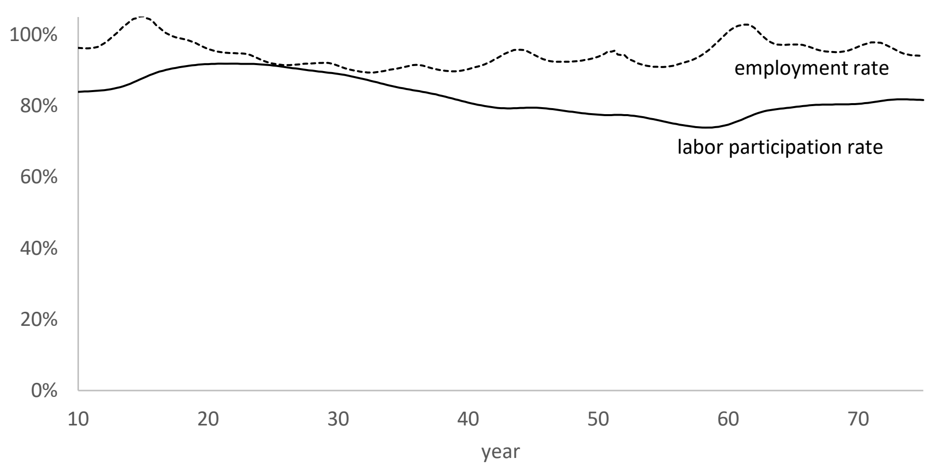
1. Labor productivity grows at a steady exponential rate;
2. The ratio of capital to the population grows at steady rate;
3. The profit share is stable;
4. The profit rate is stable;
5. The ratio of output to capital is stable;
6. The real wage grows at the same rate of labor productivity;

7. The employment ratio is stable in the long run;
8. There is a higher aggregate rate of saving out of profits than wages;
9. In the utilization-profit share plane for rich economies, there is an observed clockwise business cycle.

Some of these stylized facts are built into the model. We assume all profits are saved, while saving out of wages arises as a residual (stylized fact 8), and the Goodwin-type wage model ensures that the real wage tends to grow at the same rate as labor productivity (stylized fact 6). Others are redundant: for example, if the profit rate and capital productivity are stable (stylized facts 4 and 5), then the profit share must also be stable (stylized fact 3). We assume that firms target a steady profit rate, so the independent factor in our model is capital productivity, captured by stylized fact 5. Finally, if the output-capital ratio is steady (stylized fact 5), then the ratio of capital to the population depends on the steadiness of the GDP per capita growth rate. We therefore check stylized facts 1, 5, 7, and 9.

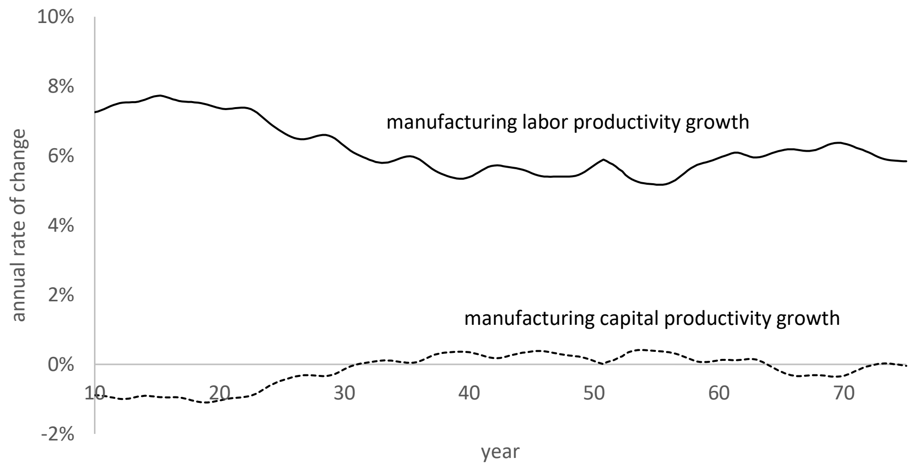
Employment and labor participation are shown in Fig. 3. The employment rate remains near 100%, with a mean of 95% and a minimum of 89%. It is maintained at a high level due to members of the working-age population entering and exiting the labor force. The participation rate varies over a long cycle, but shows no long-run trend, consistent with stylized fact 7. The participation rate has a minimum of 74% near year 60 and a maximum of 92% near year 25. This is a half-cycle of 35 years, so the full period of 70 years in this run of the model is longer than the typical period of Kondratieff (1979) long waves, which are estimated to have periods between 35 and 60 years (Metz, 1992).

Figure 3: Employment and labor participation rates



Both short and long run variation in prices, profits and wages drive changes in labor and capital productivity growth. Manufacturing productivity growth rates are shown in Fig. 4. Both labor and capital productivity growth fluctuate, but do not show a long-run trend. Labor productivity growth thus exhibits more or less steady exponential growth, consistent with stylized fact 1 as a long-run tendency. Capital productivity growth rates hover near zero, so the model exhibits a tendency towards Harrod-neutral technological change, consistent with stylized fact 5. As shown by Kemp-Benedict (2019), this is a characteristic result when cost share-induced technological change is combined with target-return pricing. Labor and capital productivity growth tend to move in opposite directions due to the cost share-induced technological change mechanism. Additionally, rising and then falling autonomous investment drives labor productivity growth through the Kaldor-Verdoorn mechanism.

Figure 4: Manufacturing labor and capital productivity growth



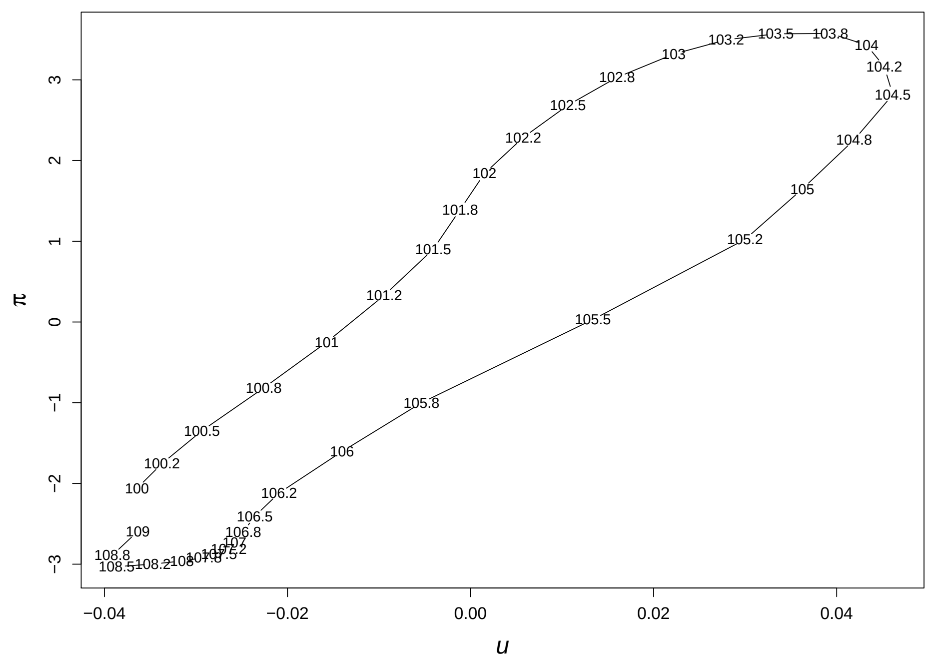
Stylized fact 9, the existence of a clockwise cycle in the (u, π) plane, was demonstrated by Nikiforos and Foley (2012). They extracted a utilization cycle from time series of log GDP and of profits from time series of the profit share of GDP. In both cases they used a Hodrick-Prescott (1997) filter, with $\lambda = 1,600$ for utilization (the standard value for quarterly data) and $\lambda = 1,000,000$ for profits, on the assumption that the functional income distribution varies comparatively slowly. We repeated their analysis with the model outputs, treating them as experimental data. The results, shown in Fig. 5, demonstrate that the model dynamics do indeed produce clockwise cycles in the (u, π) plane when measured in this way.[9]

Finally, we consider a key finding from neo-Kaleckian models, that a closed economy with a high propensity to save out of profits exhibits the paradox of thrift. Raising profits (and therefore saving) slows rather than accelerates the economy. The model presented in this paper exhibits a variety of the paradox of thrift and wage-led growth, although not in the same way as the standard neo-Kaleckian model. We present two mechanisms that exhibit this behavior.

In the model, firms target a profit rate rather than a markup. The distribution between wages and profits is dictated by technological potential through cost share-induced technological change. A rise in the profit rate initially drives the profit share up, which stimulates faster capital productivity growth and, as discussed in the Appendix, slower labor productivity growth. Slow wage growth means a slower growth in consumption expenditure,

[9]That is not the case when using average utilization and profit share as calculated in the model without smoothing. This may not be a contradiction, as Cauvel (2018) showed that the shape of the cycle is not robust against changes in variables.

Figure 5: Clockwise cycles in the (u, π) plane



which leads to lower investment and slower GDP growth. Thus, although the functional income distribution remains the same, a rising target profit rate favors capital-saving over labor-saving investment, which slows economic growth.

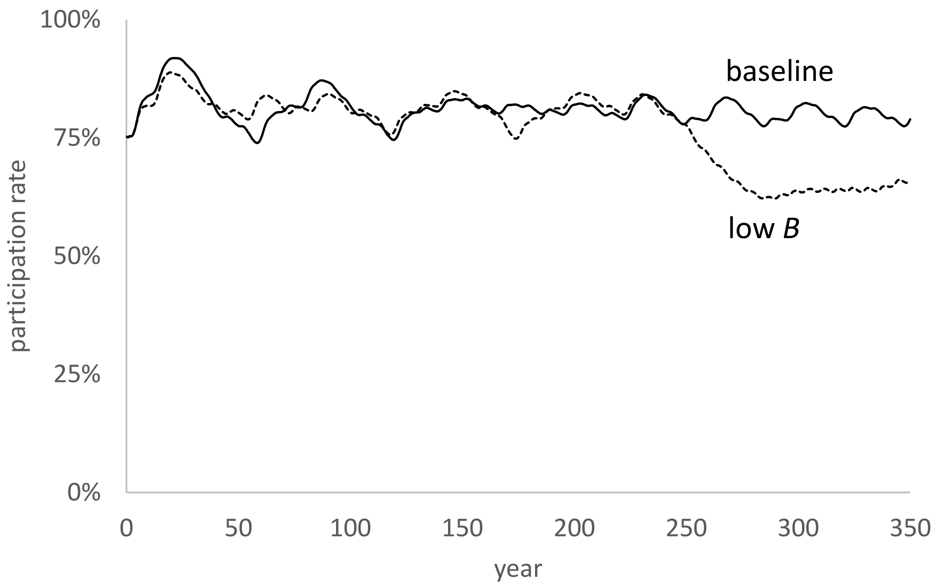
The combined system of price- and wage-setting with cost share-induced technological change features an equilibrium at which capital productivity growth is zero. The equilibrium occurs at a particular functional income distribution determined by setting $\hat{\kappa} = 0$ in Eq. (31). The equilibrium can be shifted by changing parameter b or d in that equation. In model runs, we implemented a rise in the equilibrium profit share by raising the value of parameter b[10]. As b increases, labor productivity growth briefly rises, while capital productivity growth slows. However, that pattern soon reverses as a falling wage share leads to lower consumption and slower growth – the standard Keynesian mechanism.

Finally, we show that the model presented in this paper, like that of Setterfield (2019), can produce deviations from normal business cycles, although these are uncommon. An example is shown in Fig. 6, which compares the baseline scenario to one where the "bias" term in Eqn. (19) for autonomous investment is slightly lower. Note the very long time span in the figure. The dominant cycles are "long waves" of the Kondratieff (1979) variety, with business cycles causing small deviations. Labor force participation in the two scenarios

[10]This raises the equilibrium profit share because the profit share is less than the wage share. Setting $\omega = 1 - \pi$ in Eq. (31) and imposing the equilibrium conditions that the capital productivity growth rate is zero gives $\pi = \left(1 + e^{d/b}\right)^{-1}$. This is an increasing function of b that can take values between zero and one-half.

track each other fairly closely until around year 250, where a downturn in the business cycle coincides with a downturn in the Kondratieff cycle, and the "low B" trajectory falls sharply.

Figure 6: Falling labor force participation during a depression when the bias B is not sufficiently high



5 Discussion

The model presented in this paper (and, in an earlier form, in Kemp-Benedict, 2017b; Kemp-Benedict and Ghosh, 2018) is a complex Kaleckian-Harrodian model. It is in active development, but is already producing results. In this paper we focus on what the model and its theoretical development can contribute to debates between advocates for Kaleckian and Harrodian approaches to modeling economic growth.

In broad terms, the question we asked when developing the model was: What assumptions are necessary to produce self-sustaining growth and business cycles in a multi-sectoral economy in which economic actors have only local information? Compared to most post-Keynesian models, and prompted in part by Steedman's (1992) critique, we raise the stakes by including multiple sectors and intermediate expenditure. To some degree, the economy reflected in the model is a "going concern" as argued by Means (1964). While we do not track assets, our category of "committed consumption" captures the reality that consumers take on long-term commitments by buying houses and cars, entering school, having children, paying into pensions, and so on. Firms take on long-term commitments through investment in long-lived capital stocks. The bulk of economic exchange is quite regular, but fluctuates at the margin. Firms watch for signals – excessively high or low utilization or profits – and respond to borrowing costs. Households adjust their expenditure as they are hired and laid off.

Prices are comparatively predictable, but firms gradually adjust their prices in response to costs. They adjust more or less slowly depending on overall conditions in their sector, as measured by capacity utilization, raising prices modestly when there is excess capacity and more aggressively at high utilization. Wage changes are also comparatively predictable, rising more or less in line with labor productivity growth combined with a cost of living adjustment, but slower in downturns and higher in peaks.

A necessary ingredient appears to be recurrent optimism despite periodic downturns. This is reflected in part through a sufficiently strong accelerator to drive the Harrodian instability, thus violating the Keynesian stability condition. The upward pressure periodically drives demand for labor, and therefore wages, upward as well, which is subsequently reflected in inflation. Optimism is also captured in the upward bias B in adaptive expectations for autonomous investment demand.

Despite the Harrodian instability, the model has some distinctly Kaleckian features. Most notable is that the model is closed by adjusting utilization rather than prices. As discussed earlier, prices are mostly cost-based, and change through the cost of inputs. We capture this explicitly in the case of wage costs in all sectors. We capture it implicitly in the case of land and raw materials by making prices sensitive to utilization in the sectors that depend most directly on those inputs: agriculture, mining, utilities, and construction. Nevertheless, while we do not assume a Kaldorian closure, firms can approach their target utilization and we recover some of Kaldor's stylized facts, in particular steady cost shares and capital productivity.

The model exhibits characteristically Kaleckian behavior, in that raising the profit rate or the profit share results in slower economic growth. Under target-return pricing and cost share-induced technological change, a rising profit rate leads labor productivity growth to slow, thereby reducing wages and consumption. When a higher profit share results from a change in technological potential, the change initially drives labor productivity upward, but the subsequent decline in wages and wage-dependent consumption slows demand growth.

The Sraffian super-multiplier makes an appearance in the form of quasi-autonomous non-capacity generating expenditure. First, consumption of all types of goods has a "committed" component that grows with the wage but does not fluctuate over the business cycle. Second, demand for education and health services rises proportional to GDP. These sources of demand are not fully autonomous, as indeed they cannot be. The model thus provides an illustration of how endogenous but comparatively stable demand can operate through a Sraffian super-multiplier mechanism to partially stabilize the economy, even as the Harrodian instability, bounded by a Hicksian ceiling and floor, remains a necessary driver of economic growth.

6 Conclusion

We have documented features of a multi-sector Kaleckian-Harrodian model with induced technological change. We related specific decisions in the model's development to ongoing debates over Kaleckian and Harrodian approaches to growth modeling, and showed that the model resolves Harrod's instability in the long run by endogenously bringing the actual, warranted, and natural growth rates into alignment. As with other Kaleckian-Harrodian models, we do not assume the Keynesian stability condition. The Harrodian instability is contained by a Hicksian ceiling and floor. The nonlinearity allows for asymmetric business cycles, as are actually observed. When wage and price-setting behavior are combined with induced technological change, the model gives rise to long-period waves. It features a variety of wage-led growth and can give rise to depressions.

References

Acemoglu D. (2002) 'Directed technical change', *The Review of Economic Studies*, 69(4), pp. 781–809.
URL: *http://restud.oxfordjournals.org/content/69/4/781*

Beckert J. (2016) *Imagined Futures: Fictional Expectations and Capitalist Dynamics*, Harvard University Press, Cambridge, MA, US.

Belaire-Franch J., Contreras D. (2003) 'An Assessment of International Business Cycle Asymmetries using Clements and Krolzig's Parametric Approach', *Studies in Nonlinear Dynamics & Econometrics*, 6(4).
URL: *https://www.degruyter.com/view/j/snde.2003.6.4/snde.2003.6.4.1089/snde.2003.6.4.1089.xml*

Bhaduri A., Marglin S. (1990) 'Unemployment and the real wage: The economic basis for contesting political ideologies', *Cambridge Journal of Economics*, 14(4), pp. 375–393.
URL: *http://www.jstor.org/stable/23598376*

Blatt J. M. (1978) 'On the econometric approach to business-cycle analysis', *Oxford Economic Papers*, 30(2), pp. 292–300.
URL: *http://www.jstor.org/stable/2662893*

Blatt J. M. (1980)'On the Frisch model of business cycles', *Oxford Economic Papers*, 32(3), pp. 467–479.
URL: *http://www.jstor.org/stable/2662801*

Blecker R. A. (1989)'Markup pricing, import competition, and the decline of the American steel industry', *Journal of Post Keynesian Economics*, 12(1), pp. 70–87 ArticleType: research-article / Full publication date: Autumn, 1989 / Copyright 1989 M.E. Sharpe, Inc.
URL: *http://www.jstor.org/stable/4538172*

Brock W. A. (1991)'Hicksian non-linearity', in McKenzie L. W., Zamagni S. (eds): *Value and Capital: Fifty Years Later*, International Economic Association Series, Palgrave Macmillan UK, Basingstoke, Hampshire, UK, pp. 310–330.
URL: *http://link.springer.com/chapter/10.1007/978-1-349-11029-2_19*

Cauvel M. (2018) *Three essays on the empirical estimation of wage-led and profit-led demand regimes*, Ph.D., American University, United States – District of Columbia.
URL: *https://search.proquest.com/docview/2093571856/abstract/F7EE3F9814E4C93PQ/1*

Clark T. E. (2001)'Comparing measures of core inflation', *Economic Review-Federal Reserve Bank of Kansas City*, 86(2), pp. 5–32.

Clements M. P., Krolzig H.-M. (1998) 'A comparison of the forecast performance of Markov-switching and Threshold Autoregressive models of US GNP', *The Econometrics Journal*, 1(1), pp. 47–75.
URL: *http://onlinelibrary.wiley.com/doi/10.1111/1368-423X.11004/abstract*

Corrado C., Mattey J. (1997) 'Capacity utilization', *Journal of Economic Perspectives*, 11(1), pp. 151–167.
URL: *http://www.jstor.org/stable/2138256*

Del Monte A. (1975) 'Grado di monopolio e sviluppo economico', *Rivista Internazionale di Scienze Sociali*, 46 (Anno 83)(3), pp. 231–263.
URL: *http://www.jstor.org/stable/41625003*

Duménil G., Lévy D. (1995) 'A stochastic model of technical change: An application to the US economy (1869–1989)', *Metroeconomica*, 46(3), pp. 213–245.
URL: *http://onlinelibrary.wiley.com/doi/10.1111/j.1467-999X.1995.tb00380.x/abstract*

Duménil G., Lévy D. (2010) 'The classical-Marxian evolutionary theory of technological change: application to historical tendencies', in Setterfield M. (ed.): *Handbook of Alternative Theories of Economic Growth*, Edward Elgar, Cheltenham, U.K. ; Northampton, Mass., USA, pp. 243–273.

Dutt A. K. (1984) 'Stagnation, income distribution and monopoly power', *Cambridge Journal of Economics*, 8(1), pp. 25–40.

Dutt A. K. (1987) 'Alternative closures again: A comment on'Growth, distribution and inflation'', *Cambridge Journal of Economics*, 11(1), pp. 75–82.
URL: *http://www.jstor.org/stable/23598097*

Dutt A. K. (2013) 'Endogenous technological change in Classical-Marxian models of growth and distribution', in Taylor L., Rezai A., Michl T. (eds): *Social fairness and economics: economic essays in the spirit of Duncan Foley*, Routledge, Abingdon, Oxon ; New York, NY, pp. 243–264.

Dávila-Fernández M. J., Oreiro J. L., Punzo L. F. (2019) 'Inconsistency and overdetermination in neo-Kaleckian growth models: A note', *Metroeconomica*, 70(2), pp. 313–319.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/meca.12190*

Fazzari S. M., Ferri P. E., Greenberg E. G., Variato A. M. (2013) 'Aggregate demand, instability, and growth', , 1(1), pp. 1–21.

Fazzari S. M., Ferri P., Variato A. M. G. (2018) 'Demand-Led Growth and Accommodating Supply', *SSRN Scholarly Paper ID 3108711*, Social Science Research Network, Rochester, NY.
URL: *http://papers.ssrn.com/abstract=3108711*

Fazzari S. M., Mott T. L. (1986) 'The investment theories of Kalecki and Keynes: an empirical study of firm data, 1970–1982', *Journal of Post Keynesian Economics*, 9(2), pp. 171–187.
URL: *http://www.tandfonline.com/doi/pdf/10.1080/01603477.1986.11489611*

Foley D. K. (2003) 'Endogenous technical change with externalities in a classical growth model', *Journal of Economic Behavior & Organization*, 52(2), pp. 167–189.
URL: *http://www.sciencedirect.com/science/article/pii/S0167268103000209*

Frisch R. (1933) 'Propagation problems and impulse problems in dynamic economics', *Economic Essays in Honour of Gustav Cassel*, Allen & Unwin, London, UK, pp. 171–205.

Girardi D., Pariboni R. (2019) 'Normal utilization as the adjusting variable in Neo-Kaleckian growth models: A critique', *Metroeconomica*, 70(2), pp. 341–358.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/meca.12236*

Goodwin R. M. (1951) 'The nonlinear accelerator and the persistence of business cycles', *Econometrica*, 19(1), pp. 1–17.
URL: *http://www.jstor.org/stable/1907905*

Goodwin R. M. (1967) 'A growth cycle', in Feinstein C. (ed.): *Socialism, Cpaitalism, and Economic Growth: Essays Presented to Maurice Dobb*, Cambridge University Press, Cambridge, UK, pp. 54–58.

Guo J., Lawson A. M., Planting M. A. (2002) 'From make-use to symmetric IO tables: an assessment of alternative technology assumptions', *Technical Report WP2002-3*, U.S. Department of Commerce Bureau of Economic Analysis, Cambridge, MA, US.
URL: *http://204.14.133.132/papers/pdf/alttechassump.pdf*

Harrod R. F. (1939) 'An essay in dynamic theory', *The Economic Journal*, 49(193), pp. 14–33.
URL: *http://www.jstor.org/stable/2225181*

Hein E. (2014) *Distribution and growth after Keynes*, Edward Elgar, Cheltenham, UK and Northampton, Mass., US.

Hein E., Lavoie M., Treeck T. v. (2011) 'Some instability puzzles in Kaleckian models of growth and distribution: a critical survey', *Cambridge Journal of Economics*, 35(3), pp. 587–612.
URL: *http://cje.oxfordjournals.org/content/35/3/587*

Hicks J. (1932) *The Theory of Wages*, MacMillan and Company Limited, London.

Hicks J. R. (1950) *A Contribution to the Theory of the Trade Cycle*, The Clarendon Press, Oxford, UK.

Hodrick R. J., Prescott E. C. (1997) 'Postwar U.S. business cycles: An empirical investigation', *Journal of Money, Credit and Banking*, 29(1), pp. 1–16.
URL: *http://www.jstor.org/stable/2953682*

Hymans S. H. (1972) 'Discussion of "Business cycle analysis of econommetric model simulations" by Victor Zarnowitz, Charlotte Boschan, and Geoffrey H. Moore', in Hickman B. G. (ed.): *Econometric Models of Cyclical Behavior*, Vol. 36 of *Studies in income and wealth*, Columbia University Press, New York, NY, US and London, UK.

Julius A. J. (2005) 'Steady-state growth and distribution with an endogenous direction of technical change', *Metroeconomica*, 56(1), pp. 101–125.
URL: *http://onlinelibrary.wiley.com/doi/10.1111/j.1467-999X.2005.00209.x/abstract*

Kaldor N. (1957) 'A model of economic growth', *The Economic Journal*, 67(268), pp. 591–624.
URL: *http://www.jstor.org/stable/2227704*

Kaldor N. (1961) 'Capital accumulation and economic growth', in Lutz F., Hague D. (eds): *The Theory of Capital*, Macmillan and Co. Ltd., pp. 177–222.

Kaldor N. (1966) *Causes of the Slow Rate of Economic Growth of the United Kingdom: An Inaugural Lecture*, Cambridge University Press Google-Books-ID: Mi87AAAAMAAJ.

Kemp-Benedict E. (2017a) 'Dynamic stability of post-Keynesian pricing', *The B.E. Journal of Theoretical Economics*, 17(2).
URL: *https://www.degruyter.com/view/j/bejte.ahead-of-print/bejte-2016-0013/bejte-2016-0013.xml*

Kemp-Benedict E. (2017b) 'A multi-sector Kaleckian-Harrodian model for long-run analysis', *Working Paper 1702*, Post Keynesian Economics Study Group, Leeds, UK.
URL: *https://www.postkeynesian.net/working-papers/1702/*

Kemp-Benedict E., Ghosh E. (2018) 'Downshifting in the fast lane: a post-Keynesian model of a consumer-led transition', *Economies*, 6(1).
URL: *http://www.preprints.org/manuscript/201712.0073/v1*

Kemp-Benedict E. (2019) 'Cost share-induced technological change and Kaldor's stylized facts', *Metroeconomica*, 70(1), pp. 2–23.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/meca.12223*

Kennedy C. (1964) 'Induced bias in innovation and the theory of distribution', *The Economic Journal*, 74(295), pp. 541–547.
URL: *http://www.jstor.org/stable/2228295*

Kondratieff N. D. (1979) 'The long waves in economic life', *Review (Fernand Braudel Center)*, 2(4), pp. 519–562.
URL: *http://www.jstor.org/stable/40240816*

Kurz H. D. (1986) 'Normal positions and capital utilization', *Political Economy*, 2(1), pp. 37–54.

Kurz H. D. (2010) 'Technical progress, capital accumulation and income distribution in Classical economics: Adam Smith, David Ricardo and Karl Marx', *The European Journal of the History of Economic Thought*, 17(5), pp. 1183–1222.
URL: *http://dx.doi.org/10.1080/09672567.2010.522242*

Kurz H. D., Salvadori N. (2010) 'The post-Keynesian theories of growth a distribution: a survey', in Setterfield M. (ed.): *Handbook of Alternative Theories of Economic Growth*, Edward Elgar Publishing, Cheltenham, UK, pp. 95–107 Google-Books-ID: dOIko3VeuCMC.

Lavoie M. (1995) 'The Kaleckian model of growth and distribution and its neo-Ricardian and neo-Marxian critiques', *Cambridge Journal of Economics*, 19(6), pp. 789–818.

Lavoie M. (1996) 'Mark-up pricing versus normal cost pricing in post-Keynesian models', *Review of Political Economy*, 8(1), pp. 57–66.
URL: *https://doi.org/10.1080/09538259600000035*

Lavoie M. (2014) *Post-Keynesian Economics: New Foundations*, Edward Elgar Publishing Limited, Cheltenham UK.

Lavoie M. (2015) *Introduction to Post-Keynesian Economics*, Palgrave Macmillan, Basingstoke.
URL: *http://dx.doi.org/10.1057/9780230626300*

Lavoie M. (2016) 'Convergence towards the normal rate of capacity utilization in neo-Kaleckian models: The role of non-capacity creating autonomous expenditures', *Metroeconomica*, 67(1), pp. 172–201.
URL: *http://onlinelibrary.wiley.com/doi/10.1111/meca.12109/abstract*

Lavoie M. (2019) 'Inconsistencies in the note of Dávila-Fernández, Oreiro and Punzo', *Metroeconomica*, 70(2), pp. 320–324.
URL: *https://onlinelibrary.wiley.com/doi/abs/10.1111/meca.12202*

Lavoie M., Rodríguez G., Seccareccia M. (2004) 'Similitudes and discrepancies in post-Keynesian and Marxist theories of investment: a theoretical and empirical investigation', *International Review of Applied Economics*, 18(2), pp. 127–149.
URL: *https://doi.org/10.1080/0269217042000186697*

Mattey J., Strongin S. (1997) 'Factor utilization and margins for adjusting output: evidence from manufacturing plants', *Federal Reserve Board of San Francisco Economic Review*, 1997(2).
URL: *http://www.frbsf.org/economic-research/publications/97-2/3-17-3.pdf*

Means G. (1964) *The Corporate Revolution in America*, Collier Books.

Metz R. (1992) 'A re-examination of long waves in aggregate production series', *New Findings in Long-Wave Research*, Palgrave Macmillan, London, pp. 80–119.
URL: *https://link.springer.com/chapter/10.1007/978-1-349-22450-0_4*

Minsky H. P. (2008) *John Maynard Keynes*, McGraw Hill, New York, NY, US.

Nelson R. R., Winter S. G. (1982) *An evolutionary theory of economic change*, Belknap Press of Harvard University Press, Cambridge, Mass.

Nikiforos M. (2013) 'The (normal) rate of capacity utilization at the firm level', *Metroeconomica*, 64(3), pp. 513–538.
URL: *http://onlinelibrary.wiley.com/doi/10.1111/meca.12016/abstract*

Nikiforos M. (2016) 'On the 'utilisation controversy': a theoretical and empirical discussion of the Kaleckian model of growth and distribution', *Cambridge Journal of Economics*, 40(2), pp. 437–467.
URL: *https://academic.oup.com/cje/article/40/2/437/2605022*

Nikiforos M., Foley D. K. (2012) 'Distribution and capacity utilization: Conceptual issues and empirical evidence', *Metroeconomica*, 63(1), pp. 200–229.
URL: *http://onlinelibrary.wiley.com/doi/10.1111/j.1467-999X.2011.04145.x/abstract*

Nordhaus W. D. (1973) 'Some skeptical thoughts on the theory of induced innovation', *The Quarterly Journal of Economics*, 87(2), pp. 208–219.
URL: *http://www.jstor.org/stable/1882184*

Okishio N. (1961) 'Technical changes and the rate of profit', *Kobe University Economic Review*, 7, pp. 86–99.

Palley T. (2018) 'The economics of the super-multiplier: A comprehensive treatment with labor markets', *Metroeconomica*, in press.
URL: *http://onlinelibrary.wiley.com/doi/abs/10.1111/meca.12228*

Pesaran M. H., Potter S. M. (1997) 'A floor and ceiling model of US output', *Journal of Economic Dynamics and Control*, 21(4–5), pp. 661–695.
URL: *http://www.sciencedirect.com/science/article/pii/S0165188996000024*

Razzak W. A. (2001) 'Business cycle asymmetries: International evidence', *Review of Economic Dynamics*, 4(1), pp. 230–243.
URL: *http://www.sciencedirect.com/science/article/pii/S1094202500901091*

Rowthorn R. E. (1977) 'Conflict, inflation and money', *Cambridge journal of Economics*, 1(3), pp. 215–239.
URL: *http://www.jstor.org/stable/23596632*

Rowthorn R. E. (1981) 'Demand, Real Wages and Economic Growth', *Technical Report TP/PPE/81/3*, Northeast London Polytechnic, London, UK.

Samuelson P. A. (1965) 'A theory of induced innovation along Kennedy-Weisäcker lines', *The Review of Economics and Statistics*, 47(4), pp. 343–356.
URL: *http://www.jstor.org/stable/1927763*

Serrano F. (1995a) 'Long Period Effective Demand and the Sraffian Supermultiplier', *Contributions to Political Economy*, 14(1), pp. 67–90.
URL: *http://cpe.oxfordjournals.org/content/14/1/67*

Serrano F. L. (1995b) *The Sraffian supermultiplier*, PhD Dissertation, Faculty of Economics and Politics, University of Cambridge, Cambridge, UK.

Setterfield M. (2019) 'Long-run variation in capacity utilization in the presence of a fixed normal rate', *Cambridge Journal of Economics*, 43(2), pp. 443–463.
URL: *http://academic.oup.com/cje/advance-article/doi/10.1093/cje/bey016/5055334*

Shaikh A. (2016) *Capitalism: competition, conflict, crises*, Oxford University Press, New York, NY, US.

Skott P. (2010) 'Growth, instability and cycles: Harrodian and Kaleckian models of accumulation and income distribution', in Setterfield M. (ed.): *Handbook of Alternative Theories of Economic Growth*, Edward Elgar Publishing, pp. 108–131.

Skott P. (2012) 'Theoretical and empirical shortcomings of the Kaleckian investment function', *Metroeconomica*, 63(1), pp. 109–138.
URL: *http://onlinelibrary.wiley.com/doi/10.1111/j.1467-999X.2010.04111.x/abstract*

Steedman I. (1992) 'Questions for Kaleckians', *Review of Political Economy*, 4(2), pp. 125–151.
URL: *http://www.tandfonline.com/doi/abs/10.1080/09538259200000011*

Taylor J. B. (1993) 'Discretion versus policy rules in practice', *Carnegie-Rochester Conference Series on Public Policy*, 39, pp. 195–214.
URL: *http://www.sciencedirect.com/science/article/pii/016722319390009L*

Taylor L. (1985) 'A stagnationist model of economic growth', *Cambridge Journal of Economics*, pp. 383–403.
URL: *http://www.jstor.org/stable/23597054*

Taylor L., Foley D. K., Rezai A. (2018) 'Demand drives growth all the way: Goodwin, Kaldor, Pasinetti and the Steady State', *Cambridge Journal of Economics*, .
URL: *https://academic.oup.com/cje/advance-article/doi/10.1093/cje/bey045/5250479*

Teräsvirta T., Anderson H. M. (1992) 'Characterizing nonlinearities in business cycles using smooth transition autoregressive models', *Journal of Applied Econometrics*, 7(S1), pp. S119–S136.
URL: *http://onlinelibrary.wiley.com/doi/10.1002/jae.3950070509/abstract*

Verdoorn P. J. (1949) 'Fattori che regolano lo sviluppo della produttivita del lavoro', *L'Industria*, 1, pp. 3–10 00561.

Verdoorn P. J. (2002) 'Factors that determine the growth of labour productivity (Translated by A.P. Thirlwall from the original 1949 article in Italian)', in McCombie J., Pugno M., Soro B. (eds): *Productivity Growth and Economic Performance*, Palgrave Macmillan, Basingstoke 00000.
URL: *http://www.palgraveconnect.com/doifinder/10.1057/9780230504233*

Weidman J. E., Miller K. R., Christofferson J. P., Newitt J. S. (2011) 'Best practices for dealing with price volatility in commercial construction', *International Journal of Construction Education and Research*, 7(4), pp. 276–293.
URL: *https://doi.org/10.1080/15578771.2011.552936*

A A generalized classical-evolutionary model of cost share-induced technological change

While firms in most industries cannot observe a demand curve for their output, they are not indifferent to the costs of their inputs. When the cost of one input rises, firms can be expected to favor technological innovation that saves on that input. Cost-induced technological change was a theme of the classical economists (Kurz, 2010) and it was reinvigorated by Hicks (1932). Subsequent developments led to the concept of a "technical progress function" (Kaldor, 1961; Kennedy, 1964). Samuelson (1965) recast the technical progress function in terms of a production function, but ultimately this development ran into conceptual difficulties (e.g., see Nordhaus, 1973) and was eventually abandoned. More recently, endogenous growth theories enabled Acemoglu (2002) to revisit the concept of induced technological change from a neoclassical perspective, resulting in his theory of "directed technological change", while neo-Marxian theorists have provided a range of potential models Dutt (2013).

Among the neo-Marxian theories is an evolutionary theory due to Duménil and Lévy (1995, 2010). They discarded the neoclassical assumption that firms maximize profits with knowledge of an external technological frontier in favor of an evolutionary (Nelson and Winter, 1982) assumption that firms continually perform a random search for profitable innovations in the vicinity of their current technology. Firms are assumed to make the search under conditions of fixed prices and wages, in the expectation of short-run excess profits. Thus, the Okishio (1961) viability criterion applies at the point at which they decide whether to innovate: any innovation must increase the firm's profit rate at fixed wages and prices. Subsequently, in a Marxian catch-22, those excess profits are lost as wages rise and profits fall through demands from labor to share the gains from productivity growth and from competitors driving prices downward. The process then begins again.

The evolutionary theory of Duménil and Lévy (1995, 2010) was shown by Kemp-Benedict (2019) to constrain the possible functional relationship between cost shares and productivity change. The proof is not intuitive, so we attempt to motivate it in this paper by reference to Fig. 7. Graph (a) in the figure shows coordinate axes for rates of productivity growth for labor ($\hat{\lambda}$) and capital ($\hat{\kappa}$), where the "hat" indicates a growth rate. The productivity growth rates form a vector $\hat{\boldsymbol{\nu}} = (\hat{\kappa}, \hat{\lambda})$, while the profit share (π) and wage share (ω) form a vector of cost shares $\boldsymbol{\sigma} = (\pi, \omega)$. The probability of discovering a potential innovation that would raise productivity by $\hat{\boldsymbol{\nu}}$, denoted $p(\hat{\boldsymbol{\nu}})$, is indicated by shading in the figure.

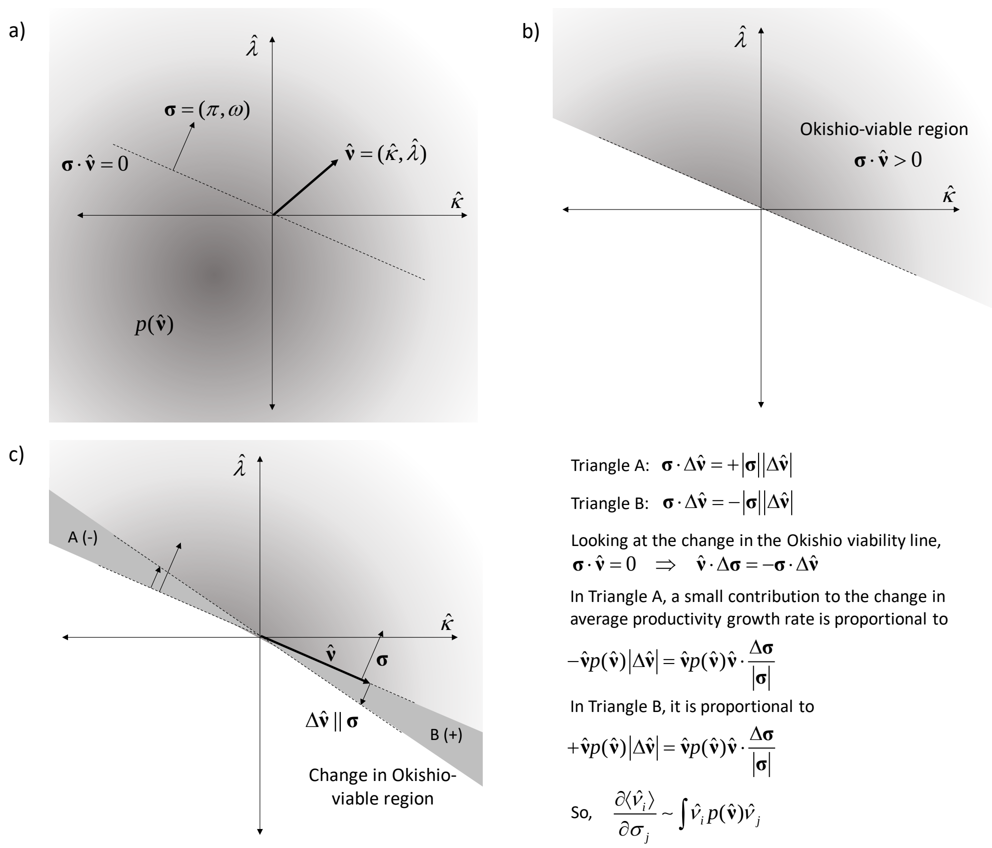
The Okishio viability criterion can be shown to imply that viable innovations must satisfy $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\nu}} > 0$ (Kemp-Benedict, 2019, p. 7). Graph (b) shows the region where the criterion is satisfied. The border of the region is the dotted line shown in Graph (a), which satisfies the condition $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\nu}} = 0$. As firms randomly discover innovations and implement those that satisfy the Okishio viability criterion, an average tendency will emerge as the expected value $\langle \hat{\boldsymbol{\nu}} \rangle$ of the productivity growth rate over the viable region where the expectation is determined by the probability of discovery $p(\hat{\boldsymbol{\nu}})$.

While the Okishio viability criterion can be stated in a compact form, the expected value of the productivity growth rate depends on the probability function $p(\hat{\boldsymbol{\nu}})$. Unless this is known – and it generally will not be – it is not possible to calculate the average productivity growth rate from first principles. However, as shown by Kemp-Benedict (2019), it is possible to constrain the functional relationship between average productivity growth and cost shares without knowing the probability distribution. Suppose that the cost shares change by a small amount $\Delta\boldsymbol{\sigma}$. The Okishio viability line with then rotate, as shown in Graph (c). While the probability distribution of discovery does not change, the location of the Okishio viability region shifts, leading to a change in the expected value of the productivity growth rate. This is a key step in the derivation, as we now explain.

The difference in the average productivity growth rate can be calculated by averaging over the small gray triangles in Graph (c). Because it is a difference, no other part of the productivity growth space enters into the calculation. Moreover, because the triangles are small (in the continuous limit, infinitesimal), we can make some approximations that simplify the calculations. The two triangles enter with opposite signs: triangle A is no longer in the Okishio viability region as a consequence of the rotation of the Okishio viability line, so it enters with a negative sign; triangle B has entered the Okishio viability region, and enters with a positive sign.

The width of the triangle grows as $\hat{\boldsymbol{\nu}}$ gets farther from the origin. In fact, as indicated by the text in Fig. 7, it is proportional to $\hat{\boldsymbol{\nu}} \cdot \Delta\boldsymbol{\sigma}$, so when calculated as the expectation of $\hat{\boldsymbol{\nu}}$ over Triangles A and B, the terms are of the form $\hat{\boldsymbol{\nu}} p(\hat{\boldsymbol{\nu}}) \hat{\boldsymbol{\nu}} \cdot \Delta\boldsymbol{\sigma}$. This means that the change in the expected value of $\hat{\nu}_i$ with respect to a change in σ_j includes terms like

Figure 7: The innovation probability distribution (indicated through a shaded density plot), the Okishio viability region, and the change in average productivity growth



$\hat{\nu}_i p(\hat{\boldsymbol{\nu}})\hat{\nu}_j$. Since the order of the terms being multiplied does not matter, this is equal to $\hat{\nu}_j p(\hat{\boldsymbol{\nu}})\hat{\nu}_i$ Each term is therefore symmetric in i and j. Moreover, the triangles in Graph (c) represent small deviations in the neighborhood of the Okishio viability line, on which $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\nu}} = 0$. To first order, we can therefore impose the approximate condition $\boldsymbol{\sigma} \cdot \hat{\boldsymbol{\nu}} \simeq 0$, so that summing the product of σ_i or σ_j with each term gives (close to) zero as the result. Finally, multiplying on the left and right by an arbitrary vector $\boldsymbol{x}$ gives a set of terms like $p(\hat{\boldsymbol{\nu}})(\boldsymbol{x} \cdot \hat{\boldsymbol{\nu}})^2$, which is positive. Taken together, these properties give the following

Result. *The matrix of partial derivatives (the Jacobian) expressing the change in the expected value of productivity with respect to a cost share is positive semi-definite, with a null vector equal to* $\boldsymbol{\sigma}$.

To recapitulate: We assume that firms continually carry out a directed but essentially random search for marginal productivity gains in the vicinity of their current technology. The expected value of the vector of productivity growth rates is taken with respect to an unknown probability of discovering an innovation that yields a particular combination of productivity growth rates over a known viability region given by the Okishio viability criterion. While the probability distribution is not known, the change in the expected value of productivity growth for input i with respect to a change in cost share j has certain properties that are independent of the probability distribution. Specifically, the matrix of partial derivatives of productivity growth with respect to cost share (the Jacobian matrix) is symmetric and positive definite. It also has a null vector given by the cost shares[11].

Before continuing, we acknowledge that the cost shares must sum to one. That is, for n inputs,

$$\sum_{i=1}^{n} \sigma_i = 1. \tag{25}$$

This condition must be applied at some point when applying this model. Indeed, they could have been applied earlier, when deriving the result about the Jacobian matrix. However, it is best not to do that, because it hides the underlying symmetry and positive semi-definiteness of the matrix. The model is easier to manipulate if the cost shares are first treated as independent and then the constraint that they sum to one is introduced at the end. We illustrate the point with an example in which there are two inputs to production.

With two inputs to production, capital and labor, the conditions derived in Kemp-Benedict (2019) and sketched above are quite constraining. Positive-definiteness implies

$$\frac{\partial\langle\hat{\kappa}\rangle}{\pi}, \frac{\partial\langle\hat{\lambda}\rangle}{\omega} > 0, \tag{26}$$

while symmetry implies

$$\frac{\partial\langle\hat{\kappa}\rangle}{\omega} = \frac{\partial\langle\hat{\lambda}\rangle}{\pi}. \tag{27}$$

Applying the null vector condition from the right then implies

$$\pi\frac{\partial\langle\hat{\kappa}\rangle}{\pi} + \omega\frac{\partial\langle\hat{\kappa}\rangle}{\omega} = 0, \tag{28}$$

$$\pi\frac{\partial\langle\hat{\lambda}\rangle}{\pi} + \omega\frac{\partial\langle\hat{\lambda}\rangle}{\omega} = 0. \tag{29}$$

Combining these conditions with the symmetry condition shows that there is only one independent partial derivative. The null vector conditions also imply that the productivity growth rates are homogeneous of order zero in the cost shares.

We use the flexibility that remains in the system of equations by proposing a formula for the cost share-induced component of labor productivity growth that is homogeneous of order zero in cost shares,

$$\hat{\lambda} = c - b\frac{\pi}{\omega}. \tag{30}$$

Then from the conditions above we find that capital productivity growth must be of the form

$$\hat{\kappa} = d + b\ln\frac{\pi}{\omega}. \tag{31}$$

These are the expressions used in the model for cost share-induced technical change. They follow from taking the expectation of productivity growth with respect to a probability of

[11] We note that the neo-Marxian models explored by Dutt (2013) do not satisfy these criteria.

discovery over a region defined by the Okishio (1961) viability criterion. While we do not know the probability distribution of discovery, the model places strong constraints on the functional relationship between productivity growth rates and cost shares. We proposed a specific functional form for labor productivity growth consistent with those constraints and then derived the functional form for capital productivity growth by applying the constraints.

Post-Keynesian models often assume that labor productivity is driven by growth rather than costs, as captured by the Kaldor-Verdoorn law (Kaldor, 1966; Verdoorn, 1949, 2002). Kemp-Benedict (2019) showed that the evolutionary theory of Duménil and Lévy (1995, 2010) is compatible with the Kaldorn-Verdoorn law. In the model, we add to Eqn. (30) a term proportional to the autonomous investment rate. Thus, labor productivity growth in the model depends on investment rates and cost shares, while capital productivity growth depends on cost shares through Eq. (31).