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A dynamic spatial model of global governance structures

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A Dynamic Spatial Model of Global Governance Structures

Abstract: This paper presents a novel understanding of the changing governance structures in global supply chains. Motivated by the global garment sector, we develop a geographical political economy dynamic model which reflects the interaction between bargaining power and distribution of value among buyer and producer firms. We find that the interplay between these two forces, in combination with the spatial specificities of global production, are necessary and sufficient to drive governance structures towards an intermediate position regarding their level of explicit coordination and power asymmetry.

Keywords: Global value chains, global production networks, uneven development, disequilibrium dynamics, monopsony power

JEL classifications: D2, R10, F6, E32, D43

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1 Introduction

The period known as 'globalization', specifically the globalizing of production and trade, brought with it new analysis and a new language. The Global Value Chains (GVC) framework played an essential role in understanding the governance of this now globalized production. GVC analysis emerged out of the Global Commodity Chain (GCC) framework of Gereffi and Korzeniewicz (1994), but went beyond the commodity by introducing governance categories that attempted to map the asymmetrical relationships between the various actors in the chain. Gereffi's (2002) original conception divided supply chains into two distinct categories of governance: *producer-driven* and *buyer-driven*, while Gereffi et al. (2005) identified five types: *market*, *modular*, *relational*, *captive* and *hierarchical*.

These frameworks have been an important means by which to analyse global production, upgrading, trade and the developmental process. Despite these successes, the GVC approach has been criticized for being static and not accounting for geographical, social and institutional specificities. These critiques have been addressed by the Global Production Networks (GPN) approach (Dicken et al. 2001; Henderson et al. 2002; Coe et al. 2004, 2008; Yeung 2009, Coe and Yeung 2015) which aims to also capture how different actors including states and institutions influence global production. GPNs place an emphasis on three variables: (i) value, (ii) power relations that affect the distribution of value and (iii) territorial embeddedness that takes account of the various social and institutional implications of global production. While GPNs added much needed complexity by including both

co-constitutive and contingent factors, this approach has been criticized for lacking the explanatory power of the GVC framework.

One of key gaps which arise due to the tension between complexity and explanatory power is the lack of a framework able to analyze the dynamics of GVC governance structures[1]. The present paper aims to fill this critical lacuna by utilizing variables from the GPN framework and introducing a formal dynamic model in which: (i) governance structures are shaped and circumscribed by the distribution of value among firms at different levels of the chain, (ii) this distribution of value depends on the bargaining power of the lead firm and (iii) this bargaining power is contingent on the geographical specificities of production. Our model, being the first of this kind, offers a novel means of studying the evolution of governance structures by taking into account the effects of geography in bargaining between global buyers (brands/retailers) and suppliers/producers (manufacturers). Our analysis draws on some, but not all, elements from both GVC and GPN literature. As such, we do not claim that our model is a formalisation of either GVC or GPN frameworks, while sharing key components of both, and thus refer to the structure of our model as a Global Value Network (GVN).

Our GVN assumes two types of firms that we call *buyers* and *producers*. Simplifying the firm types allows us to more easily link power relations with governance types. Within this context what we call *producers* are the out-sourced manufacturer sometimes referred to as 'suppliers' in the literature. Whilst what we call *buyers* includes brands, retails or even producers that

[1]For examples of changing GVC governance structures see Gereffi et al. (2005).

are 'buying' a good from an outsourced manufacturer.

We introduce the concept of Degree of Monopsony Power (DMP) as essential in determining the share of value captured by buyers in the GVN. The standard GVC typologies focus on the degree of explicit coordination of buyers to control value chains and have never focused on DMP explicitly. As we demonstrate in our model higher DMP results in greater share of value capture by buyers resulting in increased downward pressure on producers. In this way, if DMP rising then the 'buyer-driven' dynamics of the GVN are intensified. Conversely, when DMP falls then the GVN takes on a more 'producer-driven' character.

We argue that the DMP of a buyer is related to the number of producer firms able to compete for the production of a good. Due to the geographical dispersion of producers, there is a distribution of costs among producers which depends on the institutional specificities in different areas. This means that the number of firms able to produce a good will depend on the profit share of the buyer firms. Hence an increase in the profit share of the lead firms leads to a smaller number of producer firms able to compete, causing the consolidation and growth of those firms and falling DMP.

Consolidation affects the distribution process between buyer and producer firms on two levels. On a first level, consolidation reduces the number of firms which then increase their bargaining power. On a second level, consolidation allows for the cost reduction of the consolidated firm, which *ceteris paribus* leads to higher profits. We show that taking only the first level effects into account, a GVN will oscillate around what we call a "symbiotic steady state"

without converging to it, while when taking both types of effects into account the GVN will move towards the symbiotic steady state. In this way, our work highlights the importance of taking into account the various effects of consolidation. We argue that our results are in line with the empirical findings of Gereffi et al. (2005).

The formal modelling framework herein belongs to the family of the dynamic disequilibrium models frequently used in post-Keynesian and neo-Marxian traditions[2] and also has similarities with the Evolutionary Economic Geography framework[3]. The use of this type of framework to answer economic geographical questions has been persuasively argued for by Sheppard (2001), Bergmann, Sheppard and Plummer (2009), Sheppard (2011), Plummer and Sheppard and Haining (2012), among others.

The paper is structured as follows. Sections 2 and 3 motivate the key features of our modelling framework, namely allowing (i) for a continuum of governance types, (ii) for the types to change over time and (iii) to understand DMP as the principle constituent of these dynamics. Section 2 briefly examines how governance structures relate to power dynamics and demonstrates the usefulness of treating these governance structures as a continuum. Section 3, (a) builds on previous studies which reveal the dynamism of governance structures in which the specificity of relations cannot be understood through overarching categories and (b) highlights the centrality of the monopsonistic structure of the market and introduces the concept of DMP.

[2]For example, see Chiarella and Flaschel (2000), Chiarella et al. (2006), Flaschel et al. (2017) and references therein.

[3]For example see Martin, R. and P. Sunley (2007) and references therein.

In section 4, we present our formal model and our results. Section 5 concludes and poses key questions, which explore possible future extensions of this work.

2 Global Value Chain Typologies

Since the early 2000s a large part of the GVC literature has focused on the relevance of the original governance dichotomy: buyer-driven versus producer-driven. Producer-driven chains are those in which large transnational manufacturers play a central role in coordinating value chains. These chains are predominantly in high technology, vertically integrated, capital-intensive sectors such as automotive, aeronautic, and heavy machinery industries. Here value capture at the point of production is greatest because of high-barriers to entry, limited competition, and enhanced 'control over backward linkages with raw material and component suppliers, and forward linkages into distribution and retailing' (Gereffi, 2002, 3). Gereffi's theory is based on the assumption that 'lead firms' in producer-driven chains typically belong to international oligopolies (e.g. Ford, Airbus, Caterpiller, etc.). Through this oligopolistic relationship, large, often transnational firms, with access to finance and increased self-finance (through the retention of profits), are capable of substantial technological enhancement, which simultaneously increases their liability but also helps them to be dominant in the value chain, exerting a great deal of influence on smaller and highly dependent subcontracted firms.

At the other end of the spectrum we find 'buyer-driven' GVCs, which are in

low-value, low technology, vertically dis-integrated sectors such as garments, footwear, toys, furniture, and light electronics. These sectors maintain a high degree of 'fragmentation' (Arndt and Kierzkowski 2001) and a wide geographic spread. This extensive geography is an outgrowth of low barriers to entry at the producer-end since manufacturing costs are low with minimum capital investment, resulting in mostly small and mid-size firms competing intensely at the various 'lower end' phases of the GVC.

While, Gereffi's category of 'producer-driven' resembled the Fordist model of production in capitalist modernity, the novelty was found in his introduction of the buyer-driven GVC (Gibbon, Bair, Ponte 2008). A key intervention is the recognition that the relationship between buyer and producer in buyer-driven GVCs is not an equitable one, yet without a formal hierarchy. The question of power in the absence of formal hierarchy becomes a central driver in the GVC (Bair 2008). Many studies that emerged supporting the original binary, with powerful actors dictating the size, capacity, and upgrading of outsourced manufacturers (Lee and Cason 1994; Knutsen 2004; Tokatli 2004; Kumar 2014; Kumar and Mahoney 2014).

Among the most influential interventions in this debate, has been the work of Sturgeon (2001; 2002; 2003)[4]. Sturgeon (2001) builds on the original framework of Gereffi by focusing on the degree of standardisation of production and how this is reflected in the GVC. As Sturgeon (2001, 15) states, 'we need to link our terms not to firms, sectors, or places but to the specific bundles of activities that firms are engaged in.' To that end he creates the

[4]For a summary of different types of critiques see Gibbon, Bair, and Ponte (2008).

following five types of value chain/production network 'actors', defined by their 'scope of activity': integrated, retailer, lead firm, turn-key supplier, component supplier.

Integrated firms are a classically the Fordist production process and maintain high degree of vertical integration. Sturgeon's designation of *Retailer* operates at the consumer end with its scope of activity including: sales, marketing, packaging and/or system integration and includes retailers such as Gap or Wal-Mart. The *Lead Firms*, brand-name vertically dis-integrated producers who drive the GVC, and *turn- key* suppliers are firms outsourced for core functions and can include large full-package suppliers. Finally, *Component Suppliers* are lower-tier subcontractors often outsourcing secondary or periphery parts or services.

Further, in Sturgeon (2002, 2003) goes beyond simply the 'turn-key' supplier arguing for three distinct kinds of supplier firms. In addition to the turn-key, he adds commodity and captive suppliers. Again, this helps add an additional layer of clarity to a complex phenomenon. However, as with Sturgeon (2001), the focus here is based on the degree of standardisation of production. Similarly, Humphrey and Schmitz (2000, 2002) and Palpacuer (2000) emphasize supplier firm 'competency' as the distinguishing characteristic between suppliers and the power dynamics therein.

As a response to the above mentioned challenges to Gereffi's original types, Gereffi et al. (2005, 79) generate a framework to capture the 'shifting governance structures' to move beyond the duality of the buyer and producer-driven framework. In particular, they focus their attention on the 'possi-

bilities for firms in developing countries to enhance their position in global markets', and in theory we are motivated by a similar purpose. Where we part ways is in what is proposed. Gereffi et al. (2005) propose a five-part typology to value-chain governance taking account of the evolving nature of GVCs and upgrading potential. By taking account of the degree of explicit coordination and increasing power asymmetry, these governance categories are understood in the following order: market, modular, relational, captive and hierarchy.

Market GVCs have minimum costs, transactions are easily codified with simple product specifications, and both buyer and suppliers are able to switch partners with relative ease. *Modular*, similar to Sturgeon's (2002, 2003) 'turn-key', are value chains in which the supplier firm produces exclusively for lead firms with higher liability for the supplier firm. *Relational* value chains have a high degree of capability at the supplier-end contributing to a mutually dependent relationship. These value chains contain high asset specificity, and are often predicated on factors such as spatial proximity, social groups, familial links, or reputation, and have a high cost to switching. In *Captive* value chains there is a clear asymmetry (or dependent) relationship of power between small suppliers and large lead firms. In these GVCs the lead firm maintains a large degree of control. *Hierarchical* value chains are vertically integrated, with high degree of managerial and centralized control. In these value chains, the limits on finding suppliers force the lead firm to develop and manufacture in-house.

Furthermore, Gereffi et al. (2005) attempt to go beyond the buyer/producer

dichotomy by adding complexity of transactions, codifiability of information, and capability of suppliers, in determining value chain governance structures. We are indebted to their proposed typology which explains the conditions of industrial upgrading and its effect on the GVC. Our work extends on this in two directions. Firstly, while the governance categories of Gereffi et al. (2005) express a *discrete* spectrum of GVC governance characteristics, our model recognizes the spectrum as continuous. Secondly, while the aim of Gereffi et al. (2005) is to more accurately describe the characteristics of each of the categories, our focus is to understand the forces that drive the change from one structure to another and to identify any long run trend.

Both Gereffi et al. (2005) and the present paper see power relations as essential to understanding the changes in GVC structures. We both understand that there is a monotonic relationship between a variable capturing power and the degree of explicit coordination, which captures the degree to which a GVC is producer-driven. However, there is a key difference on the choice of the power variable between the two approaches. While in Gereffi et al. (2005) high degree of explicit coordination (producer-driven GVC) is related to high asymmetry of power, in our model, we focus on DMP that captures the power at the buyer-end of the chain such that high DMP is related to a buyer-driven GVC.

3 Degree of Monopsony Power in a Dynamic Context

3.1 Degree of Monopsony Power

As mentioned above, DMP is central to our model. As such we draw inspiration on one hand from Robinson's (1969) concept of monopsony in labor markets and on the other on Kalecki's (1971) 'degree of monopoly'. Monopsony according to Robinson (1969) refers to a market with many sellers and a single buyer or in the case of labor markets, a single firm and more workers than the ones needed by the firm. In the present paper we consider instead of one firm and many workers, a single buyer firm and many supplier firms. Just as an unemployed worker, competing for employment, will be willing to lower the price for their labor and/ or their conditions of work, so too, the supplier firm will be willing to lower the price for their labor and/or conditions of work.

In Kalecki (1971), the degree of monopoly of a firm, which is determined by a set of institutional factors like industrial concentration or trade union strength, defines the degree of mark up the firm puts on costs (both fixed and variable). This mark up plays a key role in the distribution of income between workers and capitalists. In our model, the degree of monopsony power affects the share of value obtained by GVN actors for given prices of output.

More concretely in our framework, we define monopsony power as the ability of the buyer firms to extract higher value from the producers than what

would be the case in a perfectly competitive market. Furthermore, the degree of monopsony power aims to capture the level of this ability, such that a relatively high degree of monopsony power results to an increase in the share of value obtained by the buyer firms. In low DMP GVNs, producers are the drivers of the GVN and tend to retain direct control over capital intensive phases of the GVN, while subcontracting out more labor-intensive functions to suppliers that are organized hierarchical and managed by the producer firm.

Within GVC analysis the concept of monopsony has been used, albeit more descriptively, to understand the asymmetry of power between buyers and producers (see Abernathy et al. 1999; Nathan et al. 2007; Milberg and Winkler 2013; Azarhoushang et al. 2015; Anner et al. 2015; Anner 2015; Mayer and Phillips 2017). We maintain that the relationship between the number of producers able to compete in relation to the number of buyers in the market shapes and circumscribes the dynamics of the GVN (see also). As Nathan and Kalpana (2007, 4) outline:

> 'The lead firms in buyer driven chains have enormous, oligopolistic market power. As buyers the volume of their purchases gives them monopsonistic power. On the other hand, with the spread of manufacturing and processing capabilities around the world, the suppliers are in very competitive markets. This asymmetry of market positions, oligopoly / monopsony vs. competitive, leads to a corresponding asymmetry in bargaining power. Lead firms are able to utilize their buying power to beat down suppliers'

prices.'

By drawing on post-Keynesian literature to get new insights in GVC related issues, our paper provides a bridge between the two approaches. In this way our work is also related to a recently expanding strand of work which uses post-Keynesian categories to enrich economic geography questions[5].

3.2 Governance Dynamics

A key aspect and contribution of our model is in recognizing that governance structures are not constant, but due to fluctuations in market power, they change over time. A prime example of this type of dynamism is found in the garment sector. The mid-1990s saw the beginning of the end for the 30-year Multifibre Agreement (MFA). Ushered in 1974, the MFA established thirty-year import quotas for garments and textiles produced in the Global South and remained one of the few checks on the unrestrained globalizing of the garment and textile sectors. The MFA period saw a particular iteration of, what David Harvey (2006) calls, the 'spatial fix' in which buyers were compelled to geographically distribute their production, resulting in export-oriented garment factories emerging in the far-flung corners of the globe (Hale and Wills 2005).

However, even under this MFA straightjacket, value nonetheless accrued to buyers sitting at the top of the chain, as the MFA came to a close this power gap between buyers and their suppliers began to widen further. Anner et

[5]For example see Milberg and Winkler (2013), Onaran and Galanis (2014), Stockhammer, Durand and Ludwig (2016) and Stockhammer (2017).

al. (2015) observe that as the MFA was phased out (1995-2005), the price paid per square meter in the international market dropped in parallel. They explain this phenomenon through two factors both linked to the end of the MFA. Firstly, buyers no longer bogged down by the same quota constraints began shifting production from regions with relatively high labor costs, such as in Mexico and Central America, towards those with lower labor costs, such as China and Southeast Asia. Secondly, they claim that these changes reflect the "growing concentration of retailer power vis-a-vis suppliers, where, as a result of monopsonistic supply chain structures, retailers and major brand manufacturers are increasingly able to squeeze lower prices from their ranks of global suppliers". In particular this latter contention corresponds with our model that shows that lowering the restrictions on trade results in higher DMP and a greater share of value to the buyer.

This coheres with Feenstra (1998) who linked the 'disintegration of production' in the international economy with the 'integration of trade'. As Gereffi et al. (2005, 80) state 'the rising integration of world markets through trade has brought with it a disintegration of multinational firms, since companies are finding it advantageous to 'outsource' an increasing share of their non-core manufacturing and service activities both domestically and abroad.'

As we moved into the post-MFA era in the mid/late 2000s, the heightened DMP reached its zenith. With increased downward pressure and falling source price offered by buyers, increasingly fewer suppliers were able to compete. This process has seen globalized competition weaken, with an almost endless number of small firms across the globe disappearing, absorbed into

larger rivals or forced to merge. What is emergent is a handful of mega-producers in a handful of labor-rich countries (Lopez-Acevedo et al. 2012; Applebaum 2008, Azmeh and Nadvi 2014, Merk 2014). Meanwhile, large retailer/brand oligopolies simultaneously benefit from growing profits brought on by economies of scale and integration, while becoming gradually dependent on increasingly oligopolistic outsourced manufacturers. Thus, with the transition away from high DMP is a move away from the 'buyer-driven' end of the spectrum.

There are number of factors contributing to this growth[6]. Critically, this is part of a logic in capitalism. As Hymer (1976, 441) states, 'Since the beginning of the industrial revolution there has been a steady increase in the size of manufacturing firms, so persistent that it might be formulated as a general law of capital accumulation.' This growth of the size of manufacturers hit a ceiling in the Global North in the economic crisis of the 1970s, resulting in a 'spatial fix', catalyzing the relentless global 'race to the bottom', a means by capital to counter a crisis of profitability. Our schema maintains that this globalizing of production increased DMP, accumulating greater shares of value for buyers, resulting in increased downward pressure on global producers.

[6]In what Nolan, Zhang and Liu (2008) label the 'cascade effect', they argue that since the 1980s there has been a growing global industrial concentration across the value chain through merger of non-core activities, which is brought on by intense downward pressure by large buyers. Nolan, Zhang and Liu (2008: 45) demonstrate that 'at every level there has taken place an intense process of industrial concentration, mainly through merger and acquisition, as firms struggle to meet the strict requirements that are the condition of their participation in the system integrators' supply chains.'

3.3 Consolidation

As part of the logic discussed above, the result of falling sourcing prices by buyers, was the vanishing of uncompetitive firms, and the mergers and acquisitions of firms into mega-firms. Here we see the emergence of an 'organisational fix', an integration of once exogenous phases in the value chain, which also places certain limits on the 'spatial fix'. Our model demonstrates the twin effects of consolidation. The first order effect is in the measure of the DMP through a reduction in the number of supplier firms. The second order effect captures the fact that a consolidated firm increases its degree of explicit coordination and is able to reduce its production costs. The combination of these twin effects mean that the share of value obtained by a lead firm is affected by the combination of these two effects.

The rise of consolidated firms in the global garment sector has a material basis in the logic of capital itself. Supplier firms in the 'buyer-driven' sector and the states they reside are placed under constant downward pressure by global buyers to cut costs, produce greater volume of goods at quicker turnover times, stock less inventory, ensure labor discipline and so on. As such what remains constant within capitalist development is the increasing efficiency, size, and reliability of firms. Over a relatively short period of time this downward pressure left fewer and fewer firms able to compete. Consequently, these firms would produce more having absorbed the production capacity of smaller firms. Marx's economic writings, particularly volume 1 and 2 of *Capital* and the *Grundrisse*, argue that the increases in speed, scale, size, and cost-efficiency, are motivated by competitive pressures and the inner logic

of capital accumulation, are thus not anomalies but part of the structural dynamics of capitalist development. Consolidation also assists in shortening the time of production, circulation, and distribution. Recent decades have seen firms associated with the 'developing world' becoming increasingly adept at generating 'value added' activities across the value chain.

There is an increased recognition of garment manufacturing firm grow, particularly in Asia, in the post-MFA world (Applebaum, 2008; Azmeh and Nadvi, 2014; Merk 2014; Gereffi 2014). The sector, whose low value, low technology, high volume, with low barriers to entry is well established, begins to change with the end of the MFA. The trend towards consolidation has been accelerated as a direct consequence of the end of the MFA. Global brands have dramatically reduced the number of suppliers in fewer countries and to reduce associated costs of logistics, warehousing, turnover time. Global buyers are now sourcing from countries that produce textiles and clothing. For example in Sri Lanka, where between 2005-2014 the number of garment factories has contracted by 50%, while the share of exports to the US and EU remained constant, and larger Sri Lankan suppliers, such as Bandix or MAS Holding, grew significantly. These are similar to trends in other 'labor-rich' countries, although legitimate data is difficult to access. Writing of this phenomenon, Merk (2014, 263) states:

> 'From the global buyer's side, purchasing departments often place the majority of their orders with a relatively small number of key suppliers. For example, 20% of contracted factories account for approximately 80% of Nike's total merchandise volume (Nike,

> 2009, p. 42). This trend towards concentration has further been accelerated by the cessation of the MFA in 2005. Many lead companies have decided to reduce the number of suppliers they use drastically and consolidate their orders in fewer countries and with fewer suppliers. To minimize logistics costs and turnover time, retailers increasingly source from countries that can produce both textiles and clothing. [...] For instance, in the years 2005-2006, sportswear company Puma eliminated 107 suppliers while Gap eliminated 615 factories (Appelbaum, 2008; Wick, 2009, p. 11). In addition, consolidation at the point of retail by companies such as Wal-Mart, Target and K-Mart also plays an important role; they prefer to place orders with large suppliers capable of handling large volumes (Hurley and Miller, 2005; UNCTAD, 2005, p. 10).

In addition, Gereffi's (2014) most recent research goes further in announcing a new phase of global governance structures whilst recognizes the growing consolidation of contractor firms such as Foxconn in electronics, Li & Fung in apparel, and Yue Yuen in footwear, with the growth of larger suppliers contributing to contracting DMP. Crucially, we identify contracting DMP as leading directly to low-value small and mid-size firms morphing into higher value large supplier firms. This move into value-added phases of the GVC transforms the decidedly buyer-driven character of the GVC into something markedly different.

3.4 Buyer Producer Symbiosis

We call this different intermediate phase as *buyer-producer symbiosis* recognizing the growing 'symbiotic' market power relationship in garment and footwear between large transnational buyers and large *transnationlizing* producers. Crucially, consolidation has played the central role in these new relations of power. As we have established GVCs are not static, and just as 'the increase[ed] disaggregation of value chains [...] allowed new kinds of lead firms to capture value' (Pickles and Smith, 2016, 25) so too the mergers, acquisition and consolidation of supplier-end capitals into large capital holding producers allow new kinds of garment and footwear supplier firms to capture value.

'Buyer-producer symbiosis', is not limited to the calculable relationship (i.e. transaction costs) between buyer and producer but resembles one-half of Gereffi's (1994, 95) original formulation which was to, "show how 'big buyers' have shaped the production networks in the world's most dynamic exporting countries, especially in the newly industrialized countries of East Asia."As Starosta (2010, 437) observes, 'the concept of governance was originally devised to depict the diversity of authority and power relationships that give overall coordination to the division of labor within the commodity chain'. In this vein, the introduction of 'symbiosis' is an observation of the power relationship, through an analysis inter alia of changes in structure, technology, and territoriality, as the consequence of the emergence of giant capitals on either side of historically low DMP GVCs.

4 Model

4.1 Setup

We consider a very basic GVN structure which consists of: one buyer firm, call this F; and N producer firms, call the i^{th} one, F^i; where $i =, 1, 2, \ldots N$. Fundamentally, firm F outsources (part of the) production to producer firms F^i. The geographical distribution of the latter, occurs unevenly and at a globally disparate scale. These conditions of production remain contingent on specific local and national cost considerations (i.e. taxes, labor laws, etc.). We assume that the costs per product c^i, of firm F^i vary over the distribution of firms.

Suppose that the end value of the product is v and let π_B be the profits of the buyer firm and π_P^i be the profits of F^i. Then the following holds

$$v = \pi_B + \pi_P^i + c^i. \tag{1}$$

Call the profit share that F obtains, $s = \frac{\pi_B}{v}$. Then (1) can be expressed alternatively as

$$s = 1 - \frac{\pi_P^i + c^i}{v}. \tag{2}$$

Given that π_P cannot be negative, (2) states that for any level of s, there exists a level of costs c^s such that only the producer firms with $c^i < c^s$ are able to compete. This means that for any level of $s \in (0, 1)$ the number of producer firms competing is $N_p < N$ with N_p decreasing with s. In this way the geographically contingent institutional differences, imply that if s is not

constant over time and the same holds for N_P.

Focusing on the profit share rather than profits has several benefits in the current framework. First, it is the appropriate variable to capture the different GVC categories. Second it is able to capture the outcomes to bargaining over the distribution of value between buyer and producer firms. Third, it allows us to abstract from issues related to (effective) demand and thus potential changes in value v which, though interesting, are out of the scope of the present paper.

Given the governance categories of Gerrefi et al. (2005), we can construct the following mapping: a relatively low N_p, implies that the governance structure of the GVN is such that the degree of explicit coordination is high. By determining the level we can then define whether the governance is *captive* or *hierarchical*. Given our continuous set up, for simplicity, we will say that low N_p and low s, means that the governance structure of the GVN is relatively producer-driven. In this way, the more producer-driven a chain - the closer it is to *hierarchical*, and the more buyer-driven - the closer it is to the *market*.

In our framework, the DMP is primarily affected by N_p, thus the DMP also changes when s changes. Let d denote the DMP, such that $d = 0$ when there is a fully competitive market. Similarly d grows mainly due to an increase of the producer firms N_p. The higher d is, the more buyer driven the governance structure of the GVN is and equivalently the lower d is the more producer-driven the structure is.

We assume that there exists a level for d, call this d^* such that for $d = d^*$, s remains unchanged, while if $d > d^*$, then s increases. This can be expressed

as

$$\dot{s} = \sigma(d - d^*), \tag{3}$$

where $\dot{s}$ is the time derivative of s and σ captures the strength of the effect of variations of d on s. In words, equation (3) argues that in the absence of other forces, (i) s will be increasing (decreasing), as long as d is higher (lower) than d^* and (ii) that the *level* of the change, depends on the distance between d and d^* and the strength constant σ. In this way, given the definition of DMP, equation (3) captures a situation where, if everything else stays the same, a small increase (decrease) of DMP relative to the d^* level is sufficient to produce to a total absorption of value by the buyer firm (producer firms).

As argued above, d depends positively on N_P which changes due to variations of s. Bringing these two together, means that an increase (decrease) in s would lead to an a decrease (increase) in d. Even though this is in accordance with what we have already discussed in the previous sections, it does not take into account the effects of consolidation among the producer firms. Consolidation which is related to the decrease in N_P, plays a key role in influencing d on two levels. On a first level, consolidation means a decrease in the total number of firms N, which also has a more long run effect to d. On a second level, consolidation means that the consolidated firms are able to reduce costs due to their increased size. As mentioned previously, we call the former effect, *first order effect* and the latter one, *second order effect*.

In order to understand the relative importance of each of these two effects more intuitively we will address them in two steps. This 'two step' approach

does not imply that it is possible to have a case with *only* first order effects, thus the isolated results due first order effects do not suggest a general conclusion for GVC dynamics.

4.2 Consolidation

4.2.1 First order effects

Assuming only the first order effect of consolidation, the change in d is given by

$$\dot{d} = \delta(s^* - s), \tag{4}$$

where s^* is the (steady state) value of s for which d stays constant and δ captures the relative importance of a variation of s on d. Equation (4) states that as long as the value of s is higher than s^*, d will keep decreasing, thus capturing the first order consolidation effect. When $s < s^*$, the opposite is true.

Equations (3) and (4) capture the power dynamics of this simple GVC. The steady state (s^*, d^*) captures a situation where both of the variables stay constant over time. This is what we can call a *buyer- producer symbiotic* or simply *symbiotic* steady state. The specific values of the s^* and d^* depend on more detailed assumptions about the effects of d and s on s and d respectively.

As the purpose of this paper is the study of the dynamics of global supply chain governance structures (out of the steady state), we have kept this specificities as general as possible. Below we show that this GVN, which

only takes into account the first level consolidation effects, exhibits power dynamics of a "predator-prey" type (Lotka, 1925; Volterra, 1926), around the symbiotic steady state[7].

Our model shows that the variables (s and d) move in a cyclical way (or like a pendulum over time) around the symbiotic steady state values (s^*, d^*)[8]. This means that that producer driven and buyer driven governance structures are two 'extreme' cases of the same framework. Furthermore, the cyclical variations around the symbiotic steady state show that:

1. As the DMP of the buyer firm becomes higher and the firm moves towards a buyer-driven state, the buyer firm is able to obtain higher share of value.

2. As the share of value obtained by the buyer firm increases, the number of firms who are able to compete becomes smaller.

3. There is a decrease in monopsony power for the buyer firm due to the decrease in the number of competing supply firms.

4. As the DMP decreases and the buyer firm moves towards the producer-driven state, the share of value it obtains becomes smaller.

5. This decrease in the share of value obtained by the buyer firm increases the number of producer firms, which in turn will increase the DMP. This brings us back to step 1.

[7]This is similar to what is observed Goodwin's (1967) 'class struggle model'.
[8]For a formal proof of this statement see Lemma 1 in the Appendix.

Figure 1 shows the intuition described above. The blue line is the share of value of the buyer firm while the red line is the DMP for $s^* = d^* = \frac{1}{2}$, $\sigma = \delta = 1$ and initial values $s(0) = d(0) = \frac{1}{3}$. The black horizontal line represents the symbiotic steady state for both s and d.

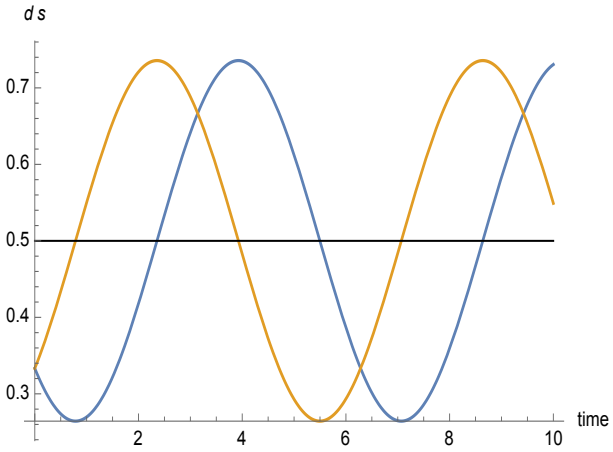

Figure 1: Buyer driven / producer driven cycles.

The graph shows us that the maximum value of both s and d is a bit over 0.7, while the minimum value is a bit less than 0.3. These values are the values which correspond to the buyer driven and producer-driven states respectively. Note that in this case the value of s at the symbiotic steady state is equal to 0.5, which means that the distance is is not equal between the symbiotic state and the extreme values of the buyer-driven and producer-driven states.

In this example we have assumed that the strength of the influence of d and s to the growth rate of s and d is the same, but this is not always the case. Below, we discuss different cases regarding the parameter values and we show that even though the shape of the graphs are different when

compared to figure 1, the underlying intuition remains the same. In this way we demonstrate that the dynamics discussed above are robust under different specifications and institutional parameters.

4.2.2 Second order effects

As we have already discussed above, consolidation does not only reduce the number of producer firms but by reducing producer firms costs, it reduces the relative effect of the bargaining power power on F on the distribution of value. This reduction is equivalent to assuming that in order to achieve the same increase in s, as without the second order effect, a higher value of d is needed. This means that by taking into account the second order effects, equation (3) becomes

$$\dot{s} = \sigma(d - \tilde{d}), \tag{5}$$

with $\tilde{d} > d^*$. Given that consolidation is not constant over time, but depends (indirectly) on s, it means that the same holds for $\tilde{d}$. As s grows the degree of competition diminishes, consolidation increases and thus $\tilde{d}$ also increases. Also, by taking into account that $\tilde{d} > d^*$, we are able to express $\tilde{d}$, in the following way

$$\tilde{d} = d^* + \beta s, \tag{6}$$

where $\beta > 0$ captures the second order effect of consolidation in relation to d^*. Then substituting (6) to (5), we are able to express the change in s in terms of d^* and s.

$$\dot{s} = \sigma(d - d^* - \beta s). \tag{7}$$

The power- value dynamics in the GVC are thus given by equations (7) and (4). Note that in this case, the new symbiotic steady state will be $s = s^*$ and $d = d^* + \beta s^*$. Then if as in the previous example $d^* = s^* = \frac{1}{2}$, the new steady state will be at $s = \frac{1}{2}$ and $d = \frac{1}{2}(1 + \beta)$.

Including the second order effects of the consolidation process makes the symbiotic steady state (asymptotically) stable[9]. This means that the power/ value dynamics of the GPN will be such that it will move towards symbiotic characteristics, no matter if it was buyer driven or producer driven. The strength of the second order effect will define whether the convergence to the symbiotic steady state will be in a cyclical manner or not[10].

We illustrate the results above with the following example. Let $s^* = d^* = \frac{1}{2}$, $\sigma = \delta = 1$ and initial values for d and s equal to $\frac{1}{3}$ as in the previous example[11]. We consider the following two cases considering the degree of the second order effects: (i) $\beta = \frac{1}{2}$ and (ii) $\beta = 2$.

[9]For a formal proof of this see Lemma 2 in the Appendix.
[10]For the relevant condition also see Lemma 2.
[11]In this case, according to Lemma 2, for a cyclical behaviour, we need $\beta < 2$.

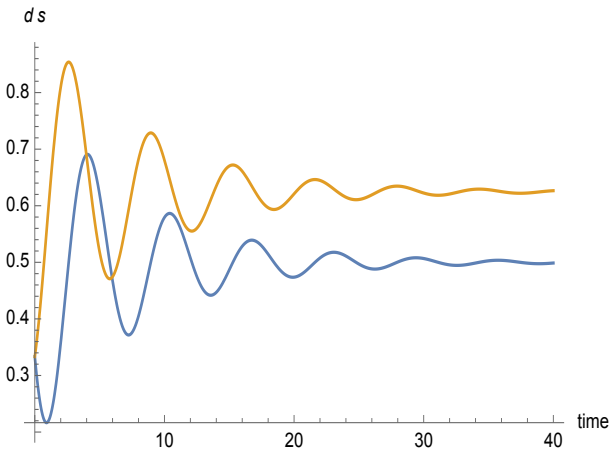

Figure 2: Low second order consolidation effects

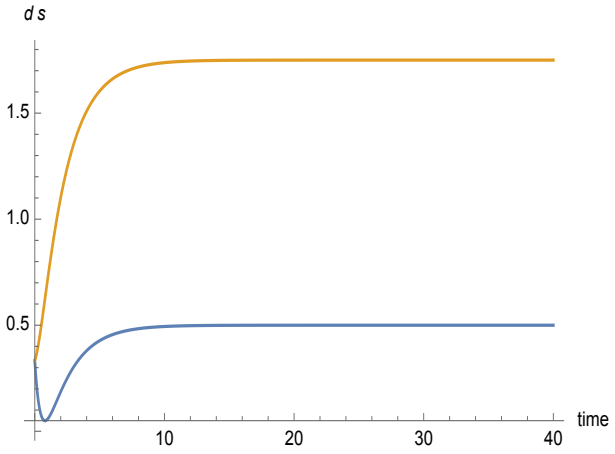

Figure 3: High second order consolidation effects.

We note that when the strength of the second order consolidation effect is low, the convergence towards the symbiotic steady state, occurs cyclically. Also notice that s initially falls before it moves towards $s^* = \frac{1}{2}$.

4.3 Case Studies

Our model shows that the convergence towards the symbiotic steady state does not depend on any assumptions regarding the relative strength of the different forces described above, thus our results are quite general and robust. It is important to highlight that the transition towards the symbiotic steady state is not necessarily a smooth one. This means that before a producer-driven GVC reaches the symbiotic steady state it may, for a relatively short period, become more buyer-driven compared to the symbiotic steady state and vice versa.

A question which naturally arises from our analysis is whether our results have empirical support. Though we have not included a specific data set informed by the theory, there are several case studies in Gereffi et al. (2005), discussed below, that illustrate how governance structures evolve over time towards 'intermediary' governance structures.

Our model reveals that, given our assumptions, a GVN converges to a symbiotic steady state. This means that the evolution of governance structures are not only towards the *market*, with low degree of explicit coordination, but also in the opposite direction. These patterns are illuminated in the case studies of Gereffi et al. (2005). In the bicycle industry we find a shift from *hierarchy* to *market* based coordination, and in garments, as we have

detailed above, the move is from *captive* to *relational* structures, while in the US electronics industry the observed change is from *hierarchy* to *modular* structures (and beyond). In all these cases, the shift is towards more buyer-driven structures with its lower degree of explicit coordination. On the other hand, the case study of fresh vegetables, from *market* to *explicit* coordination, shows the opposite shift, thus supporting our argument. Furthermore, the case studies provide some indication that the relative position of the symbiotic steady state should be close to *modular* and *relational* structures.

5 Conclusion

GVC dynamics reflect the tensions and contradictions between producers and buyers. As Inomata argues: "The main objective of GVC studies is to explore the interplay between value distribution mechanisms and organization of the cross-border production consumption nexus." (Inomata, 2017, 19). The aim of our paper has been to contribute to this objective by analyzing the governance structure dynamics. We have done this by using a simplified GVC framework where the governance structures are affected by variables used frequently in the GPN literature.

We have analysed this evolution by developing the first, to our knowledge, formal dynamic spatial model of governance structures. Despite our model drawing on both GVC and GPN research, we do not claim that ours is a general model capturing the GVC and/or GPN frameworks. In order to highlight this point, we have named our set up as GVN and we have constructed a dynamic disequilibrium model of bargaining and distribution of

value. Drawing on Kalecki (1971) and Robinson (1969), we have introduced the degree of monopsony power as the key variable that affects bargaining, such that higher DMP leads to an increase in the share of value obtained by the producer firm. The spatial specificities of production along with changes in the distribution of value, lead to consolidation and also influence DMP.

We have identified two main channels through which consolidation affects DMP. On one hand consolidation decreases the number of producer firms and decreases DMP in the short run. On the other hand, it impacts on the effectiveness of DMP in the medium run such that the rate of difficulty for the share of value to increase becomes higher with consolidation. More specifically, by accounting for the geographical specificities of both production and distribution of value, we have shown that there is an inverse relationship between DMP and producer firm profitability. This is sufficient for the GVN to oscillate between buyer and producer driven characteristics. Furthermore, for a governance structure to move towards a symbiotic state, both short and medium run effects of consolidation are necessary.

The dynamic nature of our framework highlights the importance of studying the *evolution* of governance structures. The model used here, provides a synthesis between the formal disequilibrium framework, used traditionally in neo-Marxian and post-Keynesian economics, the governance structure dynamics within the GVC framework and variables used in the GPN literature. In this way our paper bridges the traditions of formal radical political economy and economic geography. Our modelling framework also shares some common features with the evolutionary economic geography approach. In-

corporating more "evolutionary" characteristics to our model will further strengthen these links and is a possible direction for future work.

The purpose of this paper has been to examine the governance dynamics and for this reason we have kept the model general and simple. By doing this, we have necessarily kept out important factors such as the role of effective demand and technological change which could have interesting effects on governance dynamics. These omissions show possible areas of future research that our framework could be used to pursue. Our model can be extended by allowing for the effects of labor bargaining at the point of production and/or situations where there are two or more producer firms are competing, as both of these extensions could affect the bargaining process and the distribution of value, between buyer and producer firms. A different direction of research based on this work could focus in more detail on the micro dynamics of producers' costs and the process according to which new firms enter the market. Finally, given the theoretical nature of our present work, another potential related research direction is the empirical analysis associated with the governance structure dynamics in different industries.

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Appendix

Lemma 1. *The steady state of equations (3) and (4) is a stable limit cycle.*

Proof

$$\frac{d(\dot{s})}{ds} = 0$$

$$\frac{d(\dot{s})}{dd} = \sigma$$

$$\frac{d(\dot{d})}{ds} = -\delta$$

$$\frac{d(\dot{d})}{dd} = 0$$

The Jacobian matrix J is

$$J = \begin{bmatrix} 0 & \sigma \\ -\delta & 0 \end{bmatrix}$$

Let Tr, Det and Δ, denote the trace of J, determinant of J and discriminant of the characteristic polynomial of the system. Then, $Tr = 0$, $Det > 0$ and $\Delta = (Tr)^2 - 4Det < 0$. From the Routh Hurwitz conditions we know that the steady state is a stable limit cycle.

□

Lemma 2. *The steady state of equations (7) and (4) is asyptotically stable. If $\beta < 2\frac{\sqrt{\sigma\delta}}{\sigma}$ then it is a spiral node.*

Proof

$$\frac{d(\dot{s})}{ds} = -\sigma\beta$$

$$\frac{d(\dot{s})}{dd} = \sigma$$

$$\frac{d(\dot{d})}{ds} = -\delta$$

$$\frac{d(\dot{d})}{dd} = 0$$

The Jacobian matrix J is

$$J = \begin{bmatrix} -\sigma\beta & \sigma \\ -\delta & 0 \end{bmatrix}$$

Here, $Tr = -\sigma\beta$ and $Det = \sigma\delta > 0$. $\Delta < 0$ if and only if

$$(-\sigma\beta)^2 - 4\sigma\delta < 0$$

or equivalently if

$$\beta < 2\frac{\sqrt{\sigma\delta}}{\sigma}.$$

Given that $Tr < 0$ and $Det > 0$ we know from the Routh Hurwitz conditions that the steady state is asymptotically stable. In the case that $\Delta < 0$ the steady state is a spiral node.

□