abstract. In 1921 John Maynard Keynes published his famous dissertation “A Treatise On Probability” a fundamental book to understand his magnum opus “The General Theory of Employment, Interest and Money”. Although, as time goes by, his opinion about probability has rather become an inter-subjective theory; there is a concept which is fundamental for the General Theory’s framework: The weight of argument. This paper is intended to analyse a method by which one is able to compare several weights, and to explain what the argument’s meaning actually is.

Keywords: John Maynard Keynes, Methodology, Probability

Jel-Reference: B31, B41, D81
The probability

The Keynes’s theory has always been related to the concepts of uncertainty and probability, therefore, we shall embrace his most highly praised study on this matter: A Treatise on Probability. In this paper I am going to try to outline the Keynes’s probability in summary, and to focus my attention on the weight of argument.

The fundamental symbol of the Keynes’s probability is $a/h$, where $h$ is a set of propositions and $a$ is our conclusion. The probability is a logical relation between premise and conclusion, but it may not be considered to be a subjective caprice, it is objective once knowledge about circumstances is given. Therefore, if $h$ justifies a rational belief in $a$ of degree $\beta$, we say that there is a probability-relation between $a$ and $h$ of degree $\beta$, namely $a/h = \beta^1$. But, what does he mean by degree of rational belief? In order to answer to this question, we should clarify the meaning of knowledge and that of puzzles that circle around it. The first distinction is between direct and indirect knowledge; this difference is the core which rests upon the theory. The former, as Keynes stated, is that part of our rational belief which we directly know, while the latter is the part which we get to know by argument. These two parts of knowledge are related to direct acquaintance, but this lightly different from knowledge itself, in reality it arisen by acquaintance. Keynes takes into account three main classes of ways of acquisition: sensation, meaning and perception. Once we have direct acquaintance through our own sensation, we may say that it is obtained through experience. When it is obtainable through some meanings or ideas we understand it; moreover when we have direct acquaintance through meanings, characteristics, facts or relations of sense-data we can claim to have perceived it. Experience, understanding and perception are three forms of direct acquaintance$^2$; whereas we call the objects of knowledge propositions.

Now our knowledge of proposition seems to be obtained in two ways: directly, as the result of contemplating the objects of acquaintance; and indirectly, by argument, through perceiving the probability-relation of the proposition, about which we seek knowledge, to other proposition. In the second case, at any rate at first, what we know is not the proposition itself but a secondary proposition involving it...Indirect knowledge about $p$ may in suitable condition lead to rational belief in $p$ of an appropriate degree. If this degree is that of certainty, then we have not merely indirect knowledge about $p$, but indirect knowledge of $p$. (Keynes 1921, TP)

We can pass from direct to indirect knowledge, although analyzing this mental process is not always possible. This passage, from a knowledge of proposition $a$ to a knowledge about proposition $b$, comes out by perceiving a logical relation

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$^2$ibid., 12.
between them; thus we have direct acquaintance. This involves direct knowledge of the secondary proposition (Keynes 1921) and so affirms the probability-relation, furthermore this entails an indirect knowledge about or of the primary proposition. Indirect knowledge is the part of our rational belief which is based on argument, and this is very important, because, albeit a direct knowledge is the ground on which arises indirect knowledge, this one is the book’s core as well as the argument.

...whenever we pass to knowledge about one proposition by the contemplation of it in relation to another proposition of which we have knowledge...I call it an argument. (Keynes 1921, TP)

In this way truth plays an important role together with knowledge, as a matter of fact “in the case of every argument, it is only directly that we can know a secondary proposition, with which makes the argument itself valid and rational. When we know something by argument this must be through direct acquaintance with some logical relation between the conclusion and the premise”\(^3\). A proposition must be true, otherwise we cannot be aware of it, therefore knowledge relies on the true proposition, whereas direct acquaintance is the first step towards either in indirect or in direct knowledge.

Certainty is important, because the direct knowledge is certain, and this coincides with the highest degree of rational belief, so we have knowledge of proposition, while indirect knowledge leads us to a lower degree than certainty in the primary proposition. The latter is knowledge about the primary proposition. So, what is rational belief? In Keynes’s words:

There is the distinction between the part of our belief which is rational and that part which is not. If a man believes something for a reason with is preposterous or for no reason at all, and what he believes turns out to be true for some reason not know to him, he cannot be said to believe it rationally, although he believes it and it is in fact true. On the other hand, a man may rationally believe a proposition to be probable, when it is in fact false. (Keynes 1921, TP)

Probability is not caprice, it does not involve by subjective human thought, it is \textit{objective}. The argument most obviously is chosen subjectively by the human, once facts are given, probability must be considered objectively. The Treatise does not concern actual beliefs, it rather deals with rational beliefs arising from the logical relation between proposition, premise and conclusion. In order to we may have rational belief in \(p\) of a lower degree of probability than certainty, it is necessary that we know a set of propositions \(h\), and also to know some secondary propositions \(q\) asserting a probability-relation between \(p\) and \(h\)\(^4\). The objectivity is an independent logic, and Keynes links objectivity to reality, probabilistic

\(^3\)ibid., 14.
\(^4\)ibid., 16.
reasoning being viewed as the foundation of all knowledge about the world\textsuperscript{5}. Keynes unifies these two senses. Moreover if we have both certainty and direct knowledge, we will have a self-evident belief.

Hitherto Keynes seemed an empiricist, but as Rod O’Donnell remarked, he was a particular kind of rationalist\textsuperscript{6}. I agree with him, because the role of experience is necessary but insufficient. Knowledge is caused by experience; this merely directs our attention so that we are able to see its truth without requiring any proof from experience\textsuperscript{7} (Russel 1980). Therefore we need a further element: intuition. We have briefly learned, what the meaning of probability is, its objective role even if the choice of items relies upon a subjective ground. Moreover, we understood that knowledge, either direct or indirect, is yielded by experience (by direct acquaintance); \textit{ergo} means that the first proposition that we have direct acquaintance, must be true.

The Weight of Argument

The weight of argument is of utmost importance to fathom the Keynes’s theory, although he did not know its momentous role. The probability degree of an argument depends on favourable and unfavourable pieces of evidence \textsuperscript{8}, but when we talk about the weight of argument, we must think in a different way. At first, we have to state that the probability is not the same as the weight of argument (\(\zeta\)), therefore the comparison occurs between absolute amounts of relevant knowledge and of relevant ignorance respectively\textsuperscript{9}. A new piece of evidence increases \(\zeta\), and sometimes an accession of new evidence decreases the probability of an argument (\(\upsilon\)); but its weight increases just like it does in \(\zeta\). This means that \(\zeta\) and \(\upsilon\) are mismatched concepts. In Keynes’s words: 'The weight, to speak metaphorically, measures the sum of the favourable and unfavourable evidence, the probability measures the difference'; these words sum up the significance of the difference between \(\zeta \neq \upsilon\). The weight of an argument is a degree of completeness of evidence, hence an addition of a new evidence always increases its weight, but it alters the rational belief in a proposition (upwards, downwards or unchanged)\textsuperscript{10}.

\textsuperscript{6}ibid., 333.
\textsuperscript{7}ibid., 345.
\textsuperscript{9}ibid., 71.
\textsuperscript{10}We have two principal types of probability. . . in the first we compare the likelihood of two conclusion on given evidence; in the second we consider what difference a change of evidence makes to the likelihood of a given conclusion. . . (Keynes 1921, pag.54). In the first type we have the judgment preference (or indifference); in the second type we have the judgment of relevance (or indifference). In the former we compare \(x/h\) with \(y/h\), if \(x/h = y/h\) we have indifference; in the latter we compare \(x/h\) with \(x/hh_1\) and if we have equality \(x/h = x/hh_1\) we got irrelevance. Keynes gives two definitions of this judgment: the first 'That is to say, \(h_1\) is irrelevant to \(x/y\) if \(x/h_1h = x/h\)’, and the second 'That is to say, \(h_1\) is irrelevant to \(x/h\), if there is no proposition \(h'_1\) such that \(h'_1/h_1h = 1, h'_1 \neq 1\) and \(x/h'_1h \neq x/h'\).
In order to obtain a weight, we ought to compare two arguments, but it is not always possible to say of two set of proposition that one set embodies more evidence than the other (Keynes 1921, pag.77). The weight of an argument is strange to some extent, for instance, ζ embodies pieces of evidence, and sometimes ζ₁ encompasses fewer evidence than ζ. So ζ₁ > ζ can happen, because the whole information embodies in ζ is different from all evidence in ζ₁; therefore if this data is heavier than the information in ζ, ζ₁ will be used as a base of probability. This process is not always possible, because it is based on two assumptions: ⋆ Same conclusion for both ⋆ Given evidences’ framework for both. If we ignored these preconditions it would be really hard for us to assume the strength of an argument, because we must compare ζ with ζ₀. Despite of the fact that a human being has always some degree of confidence, he cannot measure the magnitude of it. Supposing that we that we can do this and that these provisos ⋆⋆ are both true; in this situation we are able to measure ζ. I take two entities into account: relevant knowledge η and relevant ignorance ξ; so the weight of an argument will be\(^{11}\):

\[
ζ = \frac{η}{η + ξ}
\]

The endless values problem was solved by this formula, and all values which we are able to measure will oscillate between weak uncertainty (\(ϵ_w=1\)) and deep uncertainty (\(ϵ_d=0\)). So we have an absolute value that will be closely to zero, but it will never be lower than zero, therefore the weight is valueless. If we have a weight that oscillates closely to one, we should only take the stochastic error into consideration, because the systematic error becomes void; therefore the weight is on its maximum degree. We often do not reach this extreme point, but our uncertainty lies between \(ϵ_w\) and \(ϵ_d\), it is called strong uncertainty (\(ϵ_s=ϵ_w ↔ ϵ_d\)). Actually it embodies the most important values which are the most frequent in everyday life.

The framework of weight encompasses both relevant knowledge and ignorance as well as it entails \(ϵ_w ← ϵ_s → ϵ_d\).

\[1\] Silvia Marzetti Dall’aste Brandolini, Roberto Scazzeri, “La Probabilità in Keynes: Premesse e Influenze” CLUEB 1999.
whereas we have another sizeable depth which is caused by one \( \eta \) in \( \zeta_2 \). Although in \( \zeta_2 \) there is only one \( \eta \), its weight is closer to \( \zeta_1 \) in which there are five elements, so \( \zeta_1 = \{ \eta_\alpha, \eta_\beta, \eta_\gamma, \xi_x, \xi_y \} \) and \( \zeta_2 = \{ \eta_\pi \} \). But in this last \( \zeta \) the elements, taken one by one, are heavier than in the first, even if its total weight is lighter than \( \zeta_1 \). In this case the choice: “which argument is the better for our conclusion” is very difficult, because it would have been erroneous to believe that \( \zeta_1 \subset \zeta_2 \); and although they have the same conclusion, they are different. An element \( \eta_\pi \) has a considerable weight itself, therefore it entails a bigger potential confidence degree in it than all elements in \( \zeta_1 \). If we add up to \( \zeta_1 \zeta_2 \) another weight \( \zeta_3 \) the difficulty increases, moreover there is \( \xi_z \) in it, and although it is lighter than \( \zeta_1 \zeta_2 \), it has sizeable weight. Which \( \zeta \) should we choose? We have:

\[
A = \begin{cases} 
\zeta_1 = \{ \eta_\alpha, \eta_\beta, \eta_\gamma, \xi_x, \xi_y \} \\
\zeta_2 = \{ \eta_\pi \} \\
\zeta_3 = \{ \xi_z \}
\end{cases}
\]

This argument \( (A) \) has three weights whose elements are different, therefore each one of them is incompatible with another \( \zeta \). This is a subjective choice and in my opinion the better picking is \( \zeta_2 \) because it has an element \( \eta_\pi \) whose weight is very significant (although \( \zeta_2 \) does not have the highest weight) and \( \zeta_2 \) is preferable to any \( \zeta \).

\[
\zeta_1 = \vee^{3(-2)}[\Delta] \quad \zeta_2 = \vee^1[\bar{\Delta}] \quad \zeta_3 = \vee^{-1}[\nabla]
\]

We must compare each weight to everything else; the above examples point out each \( \zeta \). Now, we have a “\( \vee^n \)” that is an effective weight and that depends on its own superscript \( \vee^n \): if it is negative \( \vee^{-n} \), there is a relevant ignorance \( \xi \). Moreover I listed all \( \zeta \) sorted by weight, using some symbols: \( \Delta \) is not a number, but it is the “heaviest-weight’s” symbol, \( \bar{\Delta} \) represents a weight too, but a weight which lies very closely to \( \Delta \), whilst \( \nabla \) represents a lighter weight than \( \Delta \). The square brackets mean: [ that we do not know all relevant pieces of evidence, whereas \( [\Delta] \) means that we know all available information. At this point a digression has to be made, hitherto I have illustrated that a weight of an argument concerns absolute amounts of relevant knowledge and of relevant ignorance (\( \eta \) and \( \xi \)); therefore favourable and unfavourable pieces of evidence as a whole. When we have \( \vee^{3(-2)} \) it means that 3 is an absolute relevant knowledge; 3 encompasses favourable and unfavourable pieces of evidence, for instance:

\[
\circ \circ \circ = \vee^3
\]

Each circle embodies favourable (\( f_d \)) and unfavorable (\( f_{d}^n \)) information, and its subscript \( _d \) means that this is a direct information. The minimum that all circles are able to encompasses is one \( f \), but their values strengthen when we add any favourable or unfavourable information that can be relevant. Once we have \( \vee^n \) \( (\circ \ldots \circ \) \), and it does not have irrelevant elements) a relevant knowledge appears. An argument consists also of relevant ignorance “•” and it is made
from \( f, f^u \) and at least one \( f^\xi \). The last symbol is fundamental, and it means that there is an event in which our knowledge is close to zero, but it is not zero because we have awareness of it.

\( f, f^u \) and \( f^\xi \) together create an absolute amount of \( \eta \) and \( \xi \), where:

\[
\circ = \eta \quad \bullet = \xi
\]

Each circle represents information about an event, so we can have various events, therefore several circles which encompass different favourable, unfavorable and unidentified elements. When we have irrelevant information we are able to build confidence on an \( \text{à priori} \) probability, and this is the least degree of a weight; throughout the \( \text{à priori} \) zone the weight of argument is void.

![Diagram](image)

We can compare umpteen arguments without mathematical calculus, in my opinion there are very few instances which are able to be mathematically calculated. The calculus that I have been discussing so far, is able to order every weight which cannot be calculated, and I named it transcendental calculus.

\[
\zeta_\alpha = \vee^n \Delta
\]

This is the fundamental formula, but the superscript \( n \) requires a certain knowledge about one event at least, indeed when we do not know anything about an instance and we are only aware of it, we fall in an \( \text{à priori} \) probability:

\[
\zeta_\alpha = \vee^\xi [\triangle]\]

As \( \triangle \) represents that we have the least degree of knowledge and we do not know how heavy it is, therefore \( \triangledown > \triangle \); and \( \xi \) means that we have irrelevant events, we have awareness of it.

We are able to comprehend the link between the weight of an argument and Keynes’s *General Theory* (GT) just in the preference liquidity theory and in the marginal efficiency of capital. More precisely we can fit the weight under the consumer or entrepreneur’s reason of choice and I suppose that the weight’s concept is implicit under the framework of the GT.

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\[\text{Note: The } f^\xi \text{ symbol is the necessary and sufficient condition to infer a relevant ignorance of an event. Therefore we must have } \bullet = f^\xi \text{ minimum, } \bullet = f^\xi f^\eta \text{ or } \bullet = f^\xi f^\delta \text{ etc... The fundamental piece of information is } f^\xi, \text{ all favourable elements must be always correlated to it.}\]
The type of argument which has been analyzed used the ⋆⋆ provisos, and therefore the same conclusions and given framework. In my study I have been meditating over this kind of an argument so far, and we have seen that it is capable to be compared without mathematical calculus. At this point we shall see how the procedure, as a whole, is and how I set it up. All begins with intuition (ı), thereby with direct knowledge (κd) about a secondary proposition or better from direct acquaintance (λ) about it. This is an indirect knowledge (κi) in the matter of probability because κd about a secondary proposition involves κi about (or of) a primary proposition. As we have a premise h (and its weight) which has a logical relation (β) to conclusion a; we are able to set up this process with some formal rigour:\(^\text{13}\):

\[ \Theta \rightarrow \lambda \]

\[ \iota = \lambda \in \Theta \quad \text{therefore} \quad \forall \Theta \ni \lambda \]

Different Θ involves different λ; because Θ is a set of distinct objects\(^\text{14}\) which are different ∀ Θ, and Θ ⊇ λ, Θ is a superset of λ. Once Θ and λ are given\(^\text{15}\):

\[ \hat{\lambda} \rightarrow f_d \quad \text{or} \quad f_d \nu \quad \text{or} \quad f_d \eta \]

\[ \hat{\lambda} \leftarrow \nu^n \quad \text{and} \quad \nu^n \rightarrow \zeta \quad \text{so} \quad \zeta = \nu^n \]

When there is \( \hat{\lambda} \) we have an insight which is based upon an actual fact, and therefore whatever \( f^n, \eta, \nu \) will be true. This procedure is subjective, because an intuition cannot be objective, thereby we cannot use pure mathematics because it is exact and objective.

However, we choose an argument \( \zeta_\alpha = \nu^n[\Delta] \) as we have seen above, bearing in mind that when we have an argument which is able to increase its weight through further knowledge; this one must be kept into account.

\[ \zeta_\alpha = \nu^n[\Delta] \rightarrow \kappa_d \quad \text{in a secondary proposition, and so} \quad \kappa_d \quad \text{has some} \quad \nu \]

\[ \kappa_d \rightarrow \kappa_i \quad \text{in a primary proposition} \]

All of this involves the premise\(^\text{16}\), \( \kappa_{ip} \rightarrow h \), and once we enter the field of objectivity, all procedure given, the ratio \( a/h \) becomes objective. The last ratio,

\(^{13}\) is the first element that we have awareness, Θ is whatever person who are able to perceive. An hypothetical \( x \rightarrow y \) means that there is one \( x \) which implies the existence of one \( y \), whilst when we have \( x \leftarrow y \) means that \( x \) implies \( y \) but also that \( x \) was implied by \( y \). Furthermore \( x \leftarrow y \rightarrow z \) means that \( x \) was implied by \( y \) and this one implies \( z \).

\(^{14}\) Θ encompasses several feelings, and experience, so our judgment stands in awe of this subjective process.

\(^{15}\) We can name it \( \hat{\lambda} \); and when we have \( \hat{\lambda} \rightarrow f_d \) I name it \( \hat{\lambda} \).

\(^{16}\) \( \kappa_{ip} \) is knowledge about a primary proposition.
greatly depends on the weight of an argument; this always increases its weight, while the ratio may diverge from it.

In this paper I have briefly tried to give an interpretation of the weight of argument; obviously throughout the major part of this paper I discussed according to my point of view, which may be different from that of Keynes’s.

References

