The Great Ratios in economics: A retrospective*

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Abstract

This is a preliminary draft. The belief that certain economic variables are approximately constant in the long-run is an old one. Klein & Kosobud labelled them *Great Ratios* and Kaldor described them as *Stylized Facts*. This paper sets out the history of the development of this idea, and discusses the methodological, theoretical and empirical issues involved in judging what the concept of approximately constant in the long-run means. It reviews various tests of constancy and explanation of why constancy may not hold.

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1 Introduction

The belief that certain economic variables, typically ratios, are approximately constant over the long-run is an old one. Klein & Kosobud (1961) labelled them *Great Ratios* and Kaldor (1957, 1961) described them as *Stylized Facts*. The term Great Ratios is used in the title to indicate a specific empirical concern about whether a particular set of variables are relatively stable or approximately constant over the long-run. The term stylized facts raises a set of wider methodological issues about the process of abstraction and the nature of facts in economics. These wider issues are discussed, for instance, by Boland (1987), Lawson (1989) and Abad & Khalifa (2015).

However, even this more specific empirical concern, about the long-run stability of certain variables, raises various methodological questions. Is the theoretical basis for this belief correct? What meaning should be attached to approximately constant and to the long-run? These questions are discussed below. In addition, there is a fundamental difficulty with hypothesis testing, that is known as the Duhem-Quine problem. The difficulty is that one cannot test a

*This paper has benefitted from extensive discussions with Alexander Chudik and Hashem Pesaran.

scientific hypothesis in isolation. One is nearly always testing joint hypotheses involving both the substantive hypothesis of interest, the ratio is constant, and a set of auxiliary assumptions about such things as measurement, estimation and treatment of probabilities. Thus one does not know what has been rejected, the substantive hypothesis or the auxiliary choices that are required to make the substantive hypothesis operational. For instance, the rejection may result from the lack of a sufficiently long sample for long-run tendencies to manifest themselves. Thus a failure to find a constant ratio may not prompt a reconsideration of the validity of the theory that suggests that the ratio should be constant, but instead prompt a reconsideration of the measures used or the tests or other empirical procedures that indicate the ratio is not stable.

Similarly, there are various ways that long-run stability may be defined. Klein & Kosobud treated stability as being the absence of a deterministic trend in the series. Alternatively, a stable ratio may be treated as being a unit elasticity between its components. For instance a stable debt-income ratio implies that the long run elasticity of debt to income, which can be estimated, is one. Stability has also often been interpreted in terms of the statistical definition of being stationary or more specifically being integrated of order zero, I(0), rather than having unit roots or stochastic trends, which make the series integrated of order one, I(1), stationary after being differenced once. A stable ratio is then interpreted as the difference in the logarithms of two I(1) variables, cointegrating to form an I(0) variable.

Section 2 examines the origins of the idea, since the history is quite informative; section 3 reviews the theoretical issues, including the possibility that the theory may be incorrect; section 4 reviews the empirical issues in defining long-run stability and discusses attempts to test for constancy; section 5 reviews possible reasons for instability; section 6 concludes.

2 Origins

Klein & Kosobud (1961) begin: "Economists frequently base their reasoning on key ratios between variables. If these ratios are in the nature of fundamental parameters, simplifications of theory may result. If they are simply ratios of variables, it is questionable whether any theoretical advances can be made through the transformation from statements about a quotient to statements about numerator and denominator separately." They then list some celebrated ratios in economics: the savings-income ratio, the capital-output ratio, labor's share of income, the velocity of circulation and the capital-labor ratio. They fitted linear trends to logarithms of the ratios using US data 1900-1953. and found significant trends in all but the share of wages and the participation rate. Thus, they concluded, most of these ratios did not seem to be fundamental parameters.

Kaldor (1957 p591) said "A satisfactory model concerning the nature of the growth process in a capitalist economy must also account for the remarkable historical constancies revealed by recent empirical investigations." Since some of the stylized facts are implied by others they can be presented in different ways. The most cited list is from Kaldor (1961).

"As regards the process of economic change and development in capitalist societies, I suggest the following 'stylized facts' as a starting point for the construction of theoretical models:

(1) The continued growth in the aggregate volume of production and in the productivity of labour at a steady trend rate; no recorded tendency for a *falling* rate of growth of productivity.

(2) A continued increase in the amount of capital per worker, whatever statistical measure of 'capital' is chosen in this connection.

(3) A steady rate of profit on capital, at least in the 'developed' capitalist societies; this rate of profit being substantially higher than the 'pure' long-term rate of interest as shown by the yield of gilt-edged bonds. ...

(4) Steady capital-output ratios over long periods; at least there are no clear long-term trends either rising or falling, if the differences in the degree of utilization of capacity are allowed for. This implies, or reflects, the near-identity in the percentage rates of growth of production and of capital stock - i.e. that for the economy as a whole, and over longer periods, income and capital tend to grow at the same rate.

(5) A high correlation between the share of profits in income and the share of investment in output; a steady share of profits (and of wages) in societies and/or periods in which the investment coefficient (the share of investment in output) is constant. ... The steadiness in the *share* of wages implies, of course, a rate of increase in real wages that is proportionate to the rate of growth of (average) productivity.

(6) Finally, there are appreciable differences in the *rate* of growth of labour productivity and of total output in different societies, the range of variation (in the fast growing economies) being of the order of 2-5 per cent. ..."

He then goes on to say. "None of these 'facts' can be plausibly 'explained' by the theoretical constructions of neo-classical theory. On the basis of the marginal productivity theory of Bohm-Bawerk and followers, one would expect a continued *fall* in the rate of profit with capital accumulation, and not a steady rate of profit. (In this respect classical and neo-classical theory, arguing on different grounds, come to the same conclusion - Adam Smith, Ricardo, Marx, alike with Bohm-Bawerk and Wicksell, predicted a steady fall in the rate of profit with economic progress.)"

Before giving the list he added a qualification. "Since facts, as recorded by statisticians, are always subject to numerous snags and qualifications, and for that reason are incapable of being accurately summarized, the theorist, in my view, should be free to start off with a 'stylized' view of the facts - i.e. concentrate on broad tendencies, ignoring individual detail, and proceed on the 'as if' method, i.e. construct a hypothesis that could account for these 'stylized' facts, without necessarily committing himself on the historical accuracy, or sufficiency, of the facts or tendencies thus summarized."

He argued that these low frequency historical constancies emerged, when measured over intervals longer than business cycle since they certainly fluctuated at business cycle frequencies. Kaldor (1957, p57 footnote 1) notes that in earlier work he criticised models that assumed a constant capital output ratio but accepted the validity of the assumption for long-run growth.

Although he was an extremely innovative economist, Kaldor knew little mathematics and less econometrics. With respect to mathematics, the footnote to the title of Kaldor (1957) says: "This paper owes a great deal to discussions with Professor D. G. Champernowne, both in its general ideas and even more in the detailed presentation of the mathematical parts of the argument. I am particularly indebted for his help in working out the implications of the assumptions in mathematical terms, and for the mathematical proofs of some of the propositions made, though he bears no responsibility for the choice of assumptions underlying the models." David Champernowne helped a number of Cambridge economists with their mathematics including Richard Kahn and Joan Robinson. The footnote to Kaldor (1961) says "The author is indebted to Mr. L. Pasinetti and Mr. F.H. Hahn for assistance in setting out the models in algebraic form." With respect to econometrics, Rowthorn (1975) discusses some of Kaldor's empirical procedures in a paper entitled: What remains of Kaldor's Law?

Although their lists overlapped, there were differences in the two approaches. Kaldor was interested in a theoretical growth model, Klein & Kosobud an empirical growth model. Although Klein & Kosobud had found that most of these ratios were not stable, Kaldor's stylized facts were widely adopted in theoretical models of steady state growth and the constant factor shares were used as a justification for Cobb-Douglas production functions. However, there was also some scepticism. Solow (1970, p2) commented that there "is no doubt that they are stylized, though it is possible to question whether they are facts".

Kaldor commented, partly in jest, that most economic theories rested on the combination of an accounting identity with a stylized fact. This tradition was maintained by Wyn Godley with stock-flow consistent macroeconomic models, surveyed by Nikiforos & Zezza (2018). Within this consistent accounting framework, also used by Modern Monetary Theory, the stylized facts were stock-flow norms, and deviations from these norms, such as a continuous increase in the debt to income ratio, represent unsustainable processes, but which may take a long time to reverse.

Whether facts or not, Kaldor facts have certainly been influential. Jones & Romer (2010 p225) in a paper called "The New Kaldor facts", take the old ones for granted. They say: "Redoing this exercise nearly 50 years later shows just how much progress we have made. Kaldor's first five facts have moved from research papers to textbooks. There is no longer any interesting debate about the features that a model must contain to explain them. These features are embodied in one of the great successes of growth theory in the 1950s and 1960s, the neoclassical growth model. Today, researchers are grappling with Kaldor's sixth fact and have moved on to several others that we list below." Their new facts relate to ideas, institutions, population and human capital. It is ironic that they regard Kaldor's facts as being embodied in the neo-classical growth model, whereas Kaldor regarded them as being inconsistent with that model.

Jones & Romer regard Kaldor's stylized facts as consistent with standard theory, but there are many cases where the stylised facts are inconsistent with standard theory. These are characterised as "puzzles" that prompt theoretical innovation to resolve them. Summers (1991) argues that what has contributed most to thinking about substantive issues is pragmatic empirical work, using a variety of different types of evidence, producing stylized facts, of the Kaldor type, that theory can try to explain. He cites as an example of persuasive and influential empirical work the equity premium puzzle of Mehra & Prescott (1985). They argued that the spread between the returns on stocks and bonds was inconsistent with standard theory, implying an implausibly high degree of risk aversion. This observation has proved to be a major stimulus to theory. Summers also argued that formal econometric work, where elaborate technique is used to apply theory to data or isolate the direction of causal relationships, when they are not obvious a priori, virtually always fails. Nonetheless, the focus in this paper will be on more formal econometric work.

3 Theoretical issues

3.1 The long-run

What we mean by the long-run in economics is contested, as Pesaran (1997) discusses, and there are economic and statistical definitions. In principle, the long-run represents the hypothetical state when all adjustment is complete and the system has returned to equilibrium. But that requires providing a theoretical specification of the adjustment process. Three such processes are considered below. Whether the long-run equilibrium is interesting depends on the question at hand. As Keynes noted "The long run is a misleading guide to current affairs. In the long run we are all dead. Economists set themselves too easy, too useless a task if in tempestuous seasons they can only tell us that when the storm is past the ocean is flat again."[1] However, many long-run factors have important effects that may be difficult to estimate. For instance, demographic adjustments take place over generations and have long run effects that are difficult to separate from other low frequency trends in technology and tastes. All of which may obscure the tendency towards a long-run unit elasticity for considerable periods of time.

In Keynes's example, when the storm was past, one could observe that the ocean was flat again. In real economies it may be that the storm is never over, so one needs different ways to infer what the long-run equilibrium is. But just as if one never saw a swing at rest, one could infer its resting position, because it is always tending towards it, so it is with economic data. Suppose there is a long-run equilibrium relationship, between y_t and x_t, determining an unobserved

[1] To which William Simon US Treasury secretary amid the turbulence of 1976 is said to have commented "Keynes said "In the long-run we are all dead", now Keynes is dead and we are living in the long-run."

target value of y_t, y_t^*:

$$y_t^* = \alpha + \beta x_t, \tag{1}$$

plus some adjustment process towards equilibrium. The simplest adjustment process is partial adjustment:

$$\Delta y_t = \lambda(y_t^* - y_{t-1}) + u_t.$$

Where λ is an adjustment coefficient that specifies the proportion of deviation from equilibrium that is made up in any period. This can be estimated by least squares as

$$\Delta y_t = a_0 + b x_t + a_1 y_{t-1} + u_t,$$

where $a_0 = \lambda\alpha$, $b = \lambda\beta$, $a_1 = -\lambda$. If a long run equilibrium exists, $\lambda \neq 0$, (which is testable by $H_0 : a_1 = 0$), then an estimate of the long-run coefficient can be obtained as $\widehat{\beta} = -\widehat{b}/\widehat{a}_1$, where $\widehat{b}$ and $\widehat{a}_1$ are least squares estimates. If y_t and x_t were the logarithms of the elements of a ratio, one might expect $\beta = 1$, which is testable. Often a slightly more flexible adjustment process called an error correction model is used:

$$\Delta y_t = \lambda_1 \Delta y_t^* + \lambda_2(y_{t-1}^* - y_{t-1}) + u_t.$$

This can be estimated, together with (1) by least squares as:

$$\Delta y_t = a_0 + b_0 \Delta x_t + b_1 x_{t-1} + a_1 y_{t-1} + u_t, \tag{2}$$

with $\widehat{\beta} = -\widehat{b}_0/\widehat{a}_1$. If $\lambda_2 = 0$, $b_1 = a_1 = 0$, there is no long-run relationship.

There is also a folk-wisdom among economists that cross-sections, for instance across countries, reflecting more relative adjustment and less noise can reveal long run relationships better than time-series. Thus panel data is an obvious source of information.

There are a number of theoretical mechanisms that one might expect to produce stability of such ratios, long-run unit elasticities, but none are uncontroversial and each operates over different time horizons.

3.2 Steady states

The first mechanism is the conditions required for steady state growth, the concern of Kaldor. Within the neo-classical growth model if technical progress is labour augmenting at a constant rate, a balanced growth path will involve output, investment and consumption growing at the same rate, implying constant consumption-output and investment-output ratios. Whether these are regarded as empirical facts that growth models need to match or theoretical requirements for steady state growth is sometimes not clear. One sector growth models may not be the most appropriate models to analyse the structural change associated with growth. Once one allows for multiple sectors, other ratios, such as the share of agriculture in output are clearly trending. In most industrialised countries the share of agriculture has tended to show an inverse S shaped pattern, trending downwards to a small and constant share. Such an S shaped pattern may also be true for other variables like the share of wages or the unemployment rate which are bounded by zero and one, which imposes constraints on them which strictly preclude them from being I(1) or having anything other than a unit elasticity in the (very) long run.[2]

3.3 Arbitrage

A second mechanism is arbitrage conditions, where divergence of prices creates the potential for profitable trades. This should make variables like the real exchange rate stationary because otherwise there are unexploited profit opportunities. However, risks associated with the trade and transactions costs may mean that the price divergence has to be above a threshold level before it becomes worthwhile to trade. In addition, the arbitrage conditions would apply to traded goods, while the price indices used include the non-traded services sector leading to the Samuelson-Balassa effect that consumer prices tend to be systematically higher in richer countries because of the larger difference in productivity between the traded and non-traded sectors. Because of low labour costs, services are much cheaper in poorer countries. Thus a trended real exchange rate may reflect differences in productivity growth rates.

3.4 Solvency

A third mechanism is solvency conditions which ensure that variables like balance of payments as a share of income and government deficits as a share of income are stationary to stop debt-income ratios exploding. Consider the balance of payments as an example. Let X_t be exports of goods and services, M_t, imports of goods and services, $TB_t = X_t - M_t$, the trade balance, B_t is net foreign assets, with $r_t B_t$ interest earned (paid) on foreign assets. This assumes a common interest rate, the US has benefitted from a much higher rate of return on its assets abroad than it needs to pay on its foreign borrowing. The current account is given by

$$CA_t = B_{t+1} - B_t = r_t B_t + X_t - M_t.$$

Setting $r_t = r$ (for simplicity) and solving backward from a terminal future date to the present, we have

$$B_t = \left(\frac{1}{1+r}\right)^h B_{t+h} - \sum_{\tau=0}^{h-1} \left(\frac{1}{1+r}\right)^{\tau+1} (X_{t+\tau} - M_{t+\tau}). \tag{3}$$

Let $h \to \infty$, and suppose the transversality condition, TC,

$$\lim_{h \to \infty} \left(\left(\frac{1}{1+r}\right)^h B_{t+h} \right) \tag{4}$$

[2] Testing for unit roots in bounded series raises a range of technical issues, see Cavaliere & Xu (2014).

holds, then

$$B_t = -\sum_{\tau=0}^{\infty} \left(\frac{1}{1+r} \right)^{\tau+1} (X_{t+\tau} - M_{t+\tau}). \tag{5}$$

This is the inter-temporal budget constraint, IBC. Net assets equal the negative of the (expected) present value of future trade-balances. The IBC, (5) follows from (3) only if the transversality condition (4) is satisfied. Bohm (2007) argues that the IBC is satisfied for a wide class of stochastic processes for B_t, not just difference stationary. Thus while restrictions on the order of integration are sufficient for solvency, they are not necessary. The τ period ahead conditional expectation of an mth order integrated process, $I(m)$, is at most a mth order polynomial of τ. The discounting in (4) is exponential in τ and this will dominate polynomial growth. The sufficient condition just rules out Ponzi schemes, constant new borrowing to pay old interest. He presents the sufficient condition as an error correction relationship. Suppose that $(X_t - M_t) + \alpha B_{t-1} = z_t \sim I(m)$, for some $\alpha \epsilon (0, 1+r)$, then the debt will satisfy the transversality condition (4).

While the formal analysis is similar for government debt, there are some differences. If the government issues debt in its own currency, it can inflate the debt away by printing money which it cannot do on debt denominated in a foreign currency, though countries do default on foreign debt. While a country can be a large scale lender to the rest of the world, running persistent balance of payments surpluses, it is less likely that the government is a large scale lender. Blanchard (2019) notes that the safe interest rate is currently below the growth rate, and that this has been more the historical norm than the exception. In these circumstances, public debt may have no fiscal cost, the inter-temporal government budget constraint may not bind and so there is no sustainability problem.

Of course solvency depends on expectations and lenders doubts about the borrowers ability to repay can render it insolvent. Creditworthy borrowers can borrow a lot. UK debt was over 250% of GDP at the end of Napoleonic and Second World Wars, but slowly returned to lower levels in both cases. Chudik et al. (2018) show in more recent data for a number of countries that, while log public debt and income are I(1), they do not cointegrate for around half of the countries considered. Even for those that cointegrate, there are statistically significant departures from the unit elasticity. They thus focus on short-run responses and show that the elasticity of debt with respect to output differs depending whether the rise in output is due to a fiscal shock, when it is greater than one, or to a technology shock when it is less than one.

4 Empirical Issues

Kaldor did not produce very much evidence for stability, beyond a couple of casual references. Klein and Kosobud were more systematic, fitting linear trends to logarithms of the variables using US data 1900-1953 and testing whether the trend coefficient was significant. If these series were I(1) then there is a danger of a spurious regression. In any event the standard errors from fitting a linear trend to a highly serially correlated series are likely to be wrong. There is a further problem that there may be long cycles, so that there appears to be a significant upward trend over the estimation period, but this is reversed over a longer period. The sample they used 1900-1953 may not have been long enough and was also a noisy period disturbed by two World Wars, which may have caused structural breaks.

4.1 Examples

There are now more longer historical series for more of the Great Ratios. The Bank of England has assembled a data base: a millennium of macroeconomic data for the UK, which can be used to judge stability.[3] Below is given the series on the UK share of wages from the Bank data base. The graph splices two series, A56 col. B for 1760-1860 and col. C for 1855-2016.

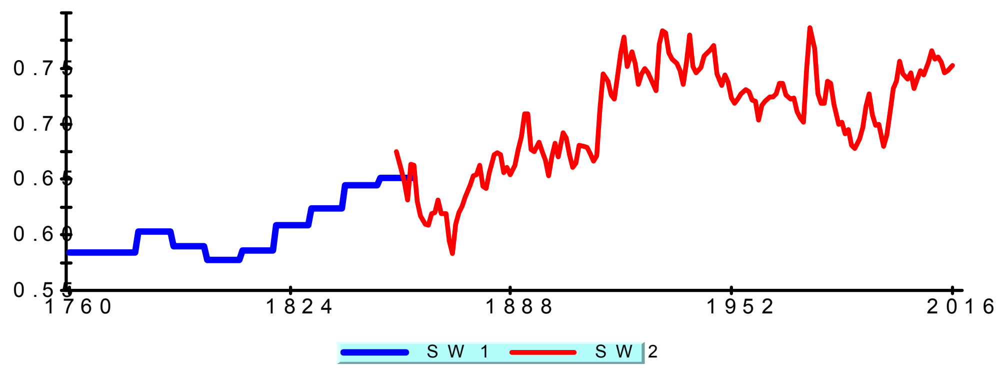

Fig 1. UK share of wages. Source Bank of England.

Whether one wishes to regard this as stable is a matter of judgement. For this and many other variables regarded as Great Ratios, there are serious problems of measurement and Klein and Kosobud emphasised that choice of numerator and denominator matters. For instance, in the case of the share of wages the treatment of the earnings of the self employed and the salaries of CEOs makes a large difference. In the case of purchasing power parity, the choice of price indices, e.g. for traded goods versus all goods in calculating the real exchange rate matters. In calculating government debt to GDP ratios the treatment of government owned assets can be important for judging solvency. For shares, real shares and nominal shares may give different answers because of relative price changes between numerator and denominator.

[3] This and a range of other historical data sources can be found at https://www.escoe.ac.uk/historical-data/.

As another example consider the real exchange rate between the US dollar and sterling from 1791, deflated using the CPI, shown in Figure 2, again taken from the Bank Millennium data base. This is broadly stable. It is somewhat more stable than the real exchange rate calculated using the GDP deflator, though both are stationary according to unit root tests. But this may be because the UK and US have broadly similar productivity growths, if one considers the real effective exchange rate in Figure 3, against a basket of currencies rather than just the dollar, there is a clear downward trend, which may be because of Samuelson-Balassa effects.

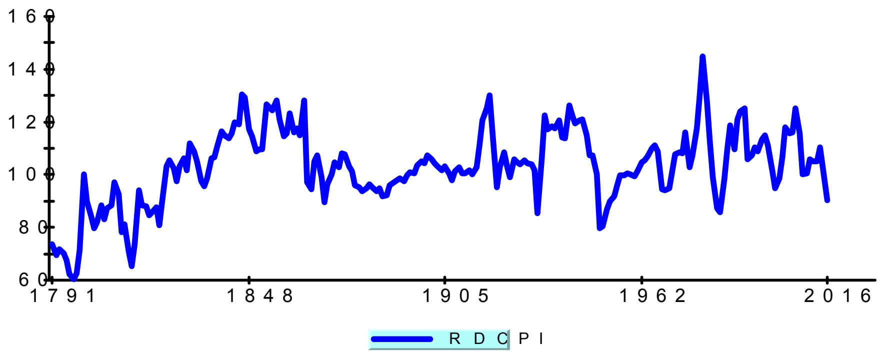

Fig 2. Real (CPI) $/£ exchange rate. Source Bank of England.

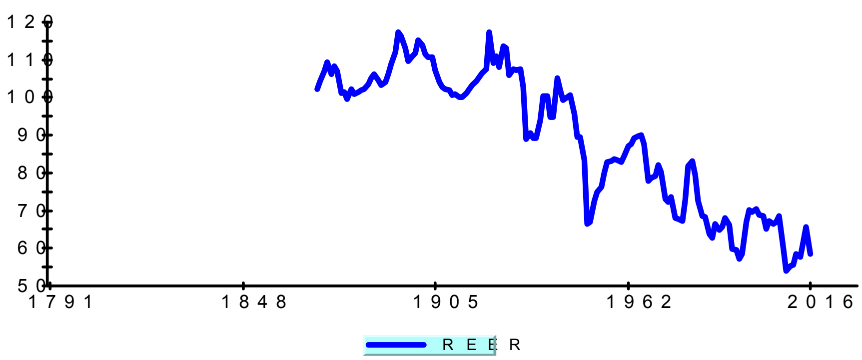

Fig3. Sterling Real Effective Exchange Rate, source Bank of England.

Of course, most investigations have gone beyond just considering graphs of the series.

4.2 Econometric representations

There are various meanings that might be attributed to constant. It might mean no deterministic trend, as Klein and Kosobud interpreted it, or a unit elasticity, or no stochastic trend and may or may not allow for level shifts. A number of level shifts are apparent in the graph of the share of wages above.

Suppose x_t and y_t are logarithms of the original components of the ratio, then in the relationship

$$z_t = y_t - \phi_t - \beta x_t + \gamma t. \tag{6}$$

the stability of the great ratio may fail because (i) no unit elasticity, $\beta \neq 1$; (ii) there is a deterministic trend, $\gamma \neq 0$; (iii) there are level shifts, ϕ_t is not constant on average or (iv) there is a stochastic trend, z_t is not $I(0)$. The random walk like behaviour when z_t is $I(1)$ may occur because there is no stabilising feedback.

This idea of stabilising feedback and long-run ratios was embodied in the error correction models of Sargan and Hendry, such as (2) above, whereby variables adjusted towards a long-run equilibrium, often the logarithm of a ratio, which acted as an attractor. Engle & Granger (1987) formalised this in terms of a cointegrating relationship. While the logarithms of the economic variables tended to be integrated of order one, I(1), stationary after being differenced once, there were linear combinations, such as the logarithm of the ratio, which were, integrated of order zero, I(0) stationary. There was a common stochastic trend in both components and this was cancelled out when the linear combination was formed. Alogoskoufis and Smith (1991) provide a history of these developments.

Suppose economic theory suggests that $\beta = 1$ and $\gamma = 0$ above, so the logarithm of great ratio, $z_t = y_t - x_t$, should be constant in the long run. Often y_t and x_t will be $I(1)$ stationary after being differenced once, then one might test if the great ratio is stationary, $I(0)$, using a unit root test for $H_0 : \alpha = 0$, against $H_1 : \alpha < 0$ in

$$\begin{aligned} \Delta z_t &= \mu + \alpha z_{t-1} + \varepsilon_t, \\ \Delta y_t - \Delta x_t &= \mu + \alpha (y_{t-1} - x_{t-1}) + \varepsilon_t. \end{aligned} \tag{7}$$

This can also be represented in a vector error correction model dropping the restriction that $\beta = 1$, as

$$\begin{aligned} \Delta y_t &= \mu_1 + \alpha_1 (y_{t-1} - \beta x_{t-1}) + \varepsilon_{1t}, \\ \Delta x_t &= \mu_2 + \alpha_2 (y_{t-1} - \beta x_{t-1}) + \varepsilon_{2t}. \end{aligned} \tag{8}$$

One can then test whether they cointegrate (a linear combination of $I(1)$ variables is $I(0)$) with a unit coefficient. Cointegration requires that at least one of α_1 or α_2 are non zero. Thus in this error correction representation, stability may fail if both the adjustment coefficients, which ensure error correction, stabilising feedback, are zero, $\alpha_1 = \alpha_2 = 0$.

The evidence that the logarithms of the Great Ratios were I(0), stationary, was not particularly strong. Pesaran & Smith (1998) note this for the logarithms of the consumption-income ratio and the investment-income ratio using the data from King et al. (1991). Harvey et al. (2003) examine four ratios for the G7 countries and find almost no evidence for stationarity of the investment-output or consumption-output ratios and mixed evidence for the real interest rate. Mills (2009) analyses the KK data using modern techniques. Tropimov (2017) considers the capital output ratio.

Among the ratios that should be stationary, most work has been done on the real exchange rate. While purchasing power parity suggests that it should be stationary, most studies do not reject the unit root hypothesis. An exception is the long span data on the real exchange rate between the dollar and sterling graphed above. Define s_t as the logarithm of the sterling dollar exchange rate and $d_t = (p_t^* - p_t)$ the logarithm of the CPI price differential. They do cointegrate and the estimated long run relationship and the error correction system for 1792-2016, corresponding to (8) is

$$\begin{array}{llllll} z_t = & s_t & \underset{(0.07)}{-0.93} & d_t & & , \\ \Delta s_t = & \underset{(0.15)}{0.36} & \underset{(0.034)}{-0.082} & z_{t-1} & +\varepsilon_{1t} & \begin{array}{l} R^2 = 0.025 \\ SER = 0.070 \end{array}, \\ \Delta d_t = & \underset{(0.12)}{0.29} & \underset{(0.028)}{+0.065} & z_{t-1} & +\varepsilon_{2t} & \begin{array}{l} R^2 = 0.023 \\ SER = 0.058 \end{array}. \end{array}$$

One cannot reject the unit elasticity hypothesis $\beta = 1$. There is also stabilising feedback both on the spot exchange rate and on the price differential, since one can reject the hypothesis that either of the $\alpha_i = 0$. The fit is poor and for many sub-periods there is not cointegration, since the adjustment coefficients are both zero. As is clear from Fig. 2, a lot of the adjustment happens after wars or major shocks, when the real exchange rate diverges substantially from its long-run value. It is possible that the real exchange rate may be I(1) within a band, set by arbitrage and transaction costs, but mean revert rapidly should it move outside that band. Pesaran et al. (2009) using pairwise tests on all possible, N(N+1)/2 real exchange rates between N=50 countries over 1957-2001, conclude that to reject the null of no adjustment to PPP requires sufficiently large disequilibria to move the real rate out of the band of inaction set by trade costs. In such cases, one can reject the null of no adjustment to PPP up to 90% of the time as compared to around 40% in the whole sample using a linear alternative and almost 60% using a non-linear alternative.

4.3 Long-run covariability

Rather than examining stationarity Muller & Watson (2018) take an alternative econometric approach to long-run covariability. They say, p775, that economic theories often have stark predictions about the covariability of variables over long-horizons, but there is a limited set of statistical tools to investigate the validity of these long-run propositions. They argue that the problems arise from the lack of long samples and the fact that inference depends on the data's long-run persistence. Different orders of integration yield statistics with different probability distributions. Inference depends critically on the exact form of long-run persistence, but there is limited sample information available to empirically determine this form. They suggest methods designed to provide reliable inference about long-run covariability for a wide range of persistence patterns. The methods rely on a relatively small number of low-frequency averages of the growth rates of the data to measure the data's long-run variability and covariability.

Muller & Watson (2018) apply their methods to balanced growth (GDP, consumption, investment, labor income, and productivity), the term structure of interest rates, the Fisher correlation (inflation and interest rates), the Phillips correlation (inflation and unemployment), PPP (exchange rates and price ratios), money growth and inflation, consumption growth and real returns, and the long-run relationship between stock prices, dividends, and earnings.

Lunsford & West (2017) use the Muller & Watson (2018) procedure on annual US data from 1890 to 2016 to investigate the long run determinants of US safe real rates and find an important role for demography and labour force growth but not GDP growth, in fact the correlation with TFP growth is negative. Del Negro et al. (2018) use the Muller & Watson (2018) procedure to examine the joint dynamics of short and long interest rates inflation and consumption for 7 now advanced economices since 1870. They find that the world real interest rate since was fairly stable around 2-2.5% till around 1980 and that it has been declining since, hitting the zero lower bound on the nominal rate.

The results in Muller & Watson (2018) and Lunsford & West (2017) suggest that in terms of estimation the Muller & Watson procedure does not add a lot over simpler methods (e.g. using decade averages) giving similar estimates. But it provides methods to do inference which are robust to the order of integration. The robustness comes at a price, the confidence intervals attached to their estimates can be very wide.

5 Reasons for non-cointegration

Were one to regard the theory for stability of the ratios as being correct and were to look for statistical reasons for its rejection, there would be many candidates. As noted above, in many cases there are serious problems of measurement. Duffy & Hendry (2017) discuss the impact of integrated measurement error on modelling long-run macroeconomic time series. If there are errors in measuring the growth rates the errors in the log level will be I(1). When there are strong trends or large shifts in the series for which the ratio is composed, they argue that cointegration analysis is not much affected by such measurement errors.

The span of the data matters many series that may appear to be I(1) over short periods but I(0) stationary over long periods, like the exchange rate example above. While the $/£ exchange rate is not stationary over shorter spans, it is over centuries. However, the UK real effective exchange rate is trended, perhaps because of Samuelson-Balassa effects. Productivity growth in the US and UK has been more similar than of the UK with other countries.

The testing can be done in a univariate manner, a unit root test on the log of the great ratio, or a multivariate manner, testing for cointegration between numerator and denominator terms. In either case inference will be sensitive to (a) the choice of null, (unit-root or no cointegration versus stationarity or cointegration) (b) the nature of the deterministic terms included: intercept, trend and any dummy variables; (c) the treatment of serial correlation, parametrically through the addition of lags or non-parametrically though the use of long run variances, with corresponding sensitivity to choice of lag length and window size; (d) how well the asymptotic critical values approximate the small sample values. Tests for a unit root null may have little power if the autoregressive coefficient is close to but not equal one. In addition changes in volatility may make detection of mean reversion more difficult. Tests for cointegration may be done in a single equation context like Dynamic OLS or fully modified OLS or in a systems context like the Johansen procedure. Different procedures may give different results. There are a very large number of estimators available and their properties differ. Stock and Watson (2017) emphasise that evidence for cointegration can be very fragile in the case of departures from exact unit roots. Small deviations from a unit root can cause large size distortions in cointegration tests.

The Univariate tests can be interpreted as reducing the dimensions of the VAR using linear transformations of the variables, where the hypothesised cointegrating vector is believed to be known *a priori* or there are natural homogeneity restrictions, e.g. using the single variable real per-capita income rather than the three variables nominal income, prices and population. Pesaran & Smith (1998) discuss these restrictions in more detail.

There may be structural breaks, regime changes and other non-linearities. Attfield & Temple (2010) point out that the equilibrium values of the great ratios depend on the structural parameters of the growth model, which may change. Thus tests for stationarity of the great ratios are testing a joint hypothesis, convergence to a balanced growth path plus the auxiliary assumption of parameter constancy. Allowing for structural breaks,.they find more evidence for the stationarity of the ratios of consumption and investment to output for the UK and US.

If the equilibrium changes there may be a long period of convergence to the new equilibrium. Siklos & Granger (1997) propose the concept of regime-sensitive cointegration whereby the variables fall in and out of an equilibrium relationship and the underlying series need not be cointegrated at all times. Cointegration is switched off when a common stochastic trend is added. There are a number of possible sources for this extra stochastic trend, or external non-stationary factor, such as technical progress, intermittent wars, natural disasters or other forms of crisis. They distinguish "time varying cointegration, in which the equilibrium relationship exists at all times, but where the underlying strength of the link is changeable or with a relationship of interest being an equilibrium one but interrupted by perhaps a one time shift to capture the advent of a new regime. Instead we suggest that economic relationships are best thought of as occasionally falling in or out of equilibrium because of some major policy shift or event." They represent this by adding a second stochastic trend

$$X_t = AW_t + \lambda_{1t}Q_t + \alpha_1 Z_t$$
$$Y_t = W_t + \lambda_{2t}Q_t + \alpha_2 Z_t$$

where $W_t, Q_t \sim I(1)$ are not cointegrated, but $Z_t \sim I(0)$, $\alpha_1 - A\alpha_2 = 1$. If $\lambda_{1t} = \lambda_{2t} = 0$, then X_t and Y_t will be cointegrated. They suggest that there can be policy changes that lead to an interfering variable which adds a stochastic trend to an existing equilibrium relationship that upsets it. They provide error correction representations for various cases.

Psaradakis et al. (2004) have Markov error-correction models where the speed of adjustment to equilibrium is different in different regimes. They apply this to the US stock-price dividend relationship discussing how this can capture periodically collapsing rational bubbles and intrinsic bubbles where the adjustment coefficients follow a two state Markov Switching model between zero adjustment and error correction. For instance the α_i in (8) may be such that the system alternates between a stabilising adjustment process and an explosive adjustment process or between a stabilising process and a zero coefficient, where the ratio becomes a random walk, perhaps with drift, or a more general I(1) process.

6 Conclusion

The discussion above emphasised analysis of a single time series. But it may be interesting to use the panel dimension and ask whether the relationships holds on average across countries. There is now long-span data in the Jorda-Schularick-Taylor Macrohistory database.[4] This has data for a range of macro and finance variables for 17 countries mainly for 1870 to 2016, T=146. This provides scope for a more extensive analysis. But the analysis does face many of the difficulties discussed above.

Overall, the empirical evidence for the stability of the great ratios is decidedly mixed. But the empirical evidence is always conditional on a set of auxiliary hypotheses about measurement and estimation methods and we do not know whether it is the substantive hypothesis of stability of the ratios or the auxiliary hypotheses that lead to rejection. In some cases, it may not be a matter of deciding whether it is the theory or the empirical methods that are wrong, but more a matter of being clear on the context. It may be sensible, in the light of the evidence, to assume the variable constant for some purposes, such as the analysis of long run growth, but not for other purposes, such as the study of cyclical processes. This is the position Kaldor took. In other cases, one needs to question all the elements - including theoretical framework, data quality and econometric techniques - to try and resolve the inconsistency.

[4] http://www.macrohistory.net/data

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